DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2022-2023)

Class : XI

MATHEMATICS

Under the Guidance of

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सत्यमेव जयते

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Message

Remembering the words of John Dewey, "Education is not preparation for life, education is life itself", I highly commend the sincere efforts of the officials and subject experts from Directorate of Education involved in the development of Support Material for classes IX to XII for the session 2022-23.

The Support Material is a comprehensive, yet concise learning support tool to strengthen the subject competencies of the students. I am sure that this will help our students in performing to the best of their abilities.

I am sure that the Heads of Schools and teachers will motivate the students to utilise this material and the students will make optimum use of this Support Material to enrich themselves.

I would like to congratulate the team of the Examination Branch along with all the Subject Experts for their incessant and diligent efforts in making this material so useful for students.

I extend my Best Wishes to all the students for success in their future endeavours.

(Ashok Kumar)

HIMANSHU GUPTA, IAS
Director, Education & Sports

सत्यमेव जयते

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MESSAGE

"A good education is a foundation for a better future."

- Elizabeth Warren

Believing in this quote, Directorate of Education, GNCT of Delhi tries to fulfill its objective of providing quality education to all its students.

Keeping this aim in mind, every year support material is developed for the students of classes IX to XII. Our expert faculty members undertake the responsibility to review and update the Support Material incorporating the latest changes made by CBSE. This helps the students become familiar with the new approaches and methods, enabling them to become good at problem solving and critical thinking. This year too, I am positive that it will help our students to excel in academics.

The support material is the outcome of persistent and sincere efforts of our dedicated team of subject experts from the Directorate of Education. This Support Material has been especially prepared for the students. I believe its thoughtful and intelligent use will definitely lead to learning enhancement.

Lastly, I would like to applaud the entire team for their valuable contribution in making this Support Material so beneficial and practical for our students.

Best wishes to all the students for a bright future.

(HIMANSHU GUPTA)

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सत्यमेव जयते

संदेश

शिक्षा निदेशालय, दिल्ली सरकार का महत्वपूर्ण लक्ष्य अपने विद्यार्थियों का सर्वांगीण विकास करना है। इस उद्देश्य को ध्यान में रखते हुए शिक्षा निदेशालय ने अपने विद्यार्थियों को उच्च कोटि के शैक्षणिक मानकों के अनुरूप विद्यार्थियों के स्तरानुकूल सहायक सामग्री उपलब्ध कराने का प्रयास किया है। कोरोना काल के कठिनतम समय में भी शिक्षण अधिगम की प्रक्रिया को निर्बाध रूप से संचालित करने के लिए संबंधित समस्त अकादमिक समूहों और क्रियान्वित करने वाले शिक्षकों को हार्दिक बधाई देती हूँ।

प्रत्येक वर्ष की भाँति इस वर्ष भी कक्षा 9वीं से कक्षा 12वीं तक की सहायक सामग्रियों में सी.बी.एस.ई. के नवीनतम दिशा-निर्देशों के अनुसार पाठ्यक्रम में आवश्यक संशोधन किए गए हैं। साथ ही साथ मूल्यांकन से संबंधित आवश्यक निर्देश भी दिए गए हैं। इन सहायक सामग्रियों में कठिन से कठिन पाठ्य सामग्री को भी सरलतम रूप में प्रस्तुत किया गया है ताकि शिक्षा निदेशालय के विद्यार्थियों को इसका भरपूर लाभ मिल सके।

मुझे आशा है कि इन सहायक सामग्रियों के गहन और निरंतर अध्ययन के फलस्वरूप विद्यार्थियों में गुणात्मक शैक्षणिक संवर्धन का विस्तार उनके प्रदर्शनो में भी परिलक्षित होगा। इस उत्कृष्ट सहायक सामग्री को तैयार करने में शामिल सभी अधिकारियों तथा शिक्षकों को हार्दिक बधाई देती हूँ तथा सभी विद्यार्थियों को उनके उज्ज्वल भविष्य की शुभकामनाएं देती हूँ।

रीता शर्मा

(रीता शर्मा)

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक [1]**[संपूर्ण प्रभुत्व-संपन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य]** बनाने के लिए, तथा उसके समस्त नागरिकों को **:**

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की **समता**

प्राप्त कराने के लिए,

तथा उन **सब में**

व्यक्ति की गरिमा और [2]**[राष्ट्र की एकता और अखंडता]** सुनिश्चित करने वाली **बंधुता**

बढ़ाने के लिए

दृढ़संकल्प होकर **अपनी इस संविधान सभा में** आज तारीख 26 नवंबर, 1949 ई. को **एतद्द्वारा इस संविधान को अंगीकृत, अधिनियमित और आत्मार्पित करते हैं।**

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1. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''प्रभुत्व-संपन्न लोकतंत्रात्मक गणराज्य'' के स्थान पर प्रतिस्थापित।
2. संविधान (बयालीसवां संशोधन) अधिनियम, 1976 की धारा 2 द्वारा (3.1.1977 से) ''राष्ट्र की एकता'' के स्थान पर प्रतिस्थापित।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

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1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51 क

मूल कर्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

(क) संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्रध्वज और राष्ट्रगान का आदर करे;

(ख) स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे;

(ग) भारत की संप्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण बनाए रखे;

(घ) देश की रक्षा करे और आह्वान किए जाने पर राष्ट्र की सेवा करे;

(ङ) भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे जो धर्म, भाषा और प्रदेश या वर्ग पर आधारित सभी भेदभावों से परे हो, ऐसी प्रथाओं का त्याग करे जो महिलाओं के सम्मान के विरुद्ध हों;

(च) हमारी सामासिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका परिरक्षण करे;

(छ) प्राकृतिक पर्यावरण की, जिसके अंतर्गत वन, झील, नदी और वन्य जीव हैं, रक्षा करे और उसका संवर्धन करे तथा प्राणिमात्र के प्रति दयाभाव रखे;

(ज) वैज्ञानिक दृष्टिकोण, मानववाद और ज्ञानार्जन तथा सुधार की भावना का विकास करे;

(झ) सार्वजनिक संपत्ति को सुरक्षित रखे और हिंसा से दूर रहे;

(ञ) व्यक्तिगत और सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे, जिससे राष्ट्र निरंतर बढ़ते हुए प्रयत्न और उपलब्धि की नई ऊँचाइयों को छू सके; और

(ट) यदि माता-पिता या संरक्षक है, छह वर्ष से चौदह वर्ष तक की आयु वाले अपने, यथास्थिति, बालक या प्रतिपाल्य को शिक्षा के अवसर प्रदान करे।

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

DIRECTORATE OF EDUCATION

GOVT. of NCT, DELHI

SUPPORT MATERIAL

(2022-2023)

MATHEMATICS

CLASS : XI

NOTE FOR SALE

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2022-2023

SUPPORT MATERIAL

Class : XI

MATHEMATICS

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Mathematics (XI-XII)
(Code No. 041)
Session - 2022-23

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. Senior Secondary stage is a launching stage from where the students go either for higher academic education in mathematics or for professional course like Engineering, Physical and Biological Science, Commerce or Computer Applications. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in Focus Group on Teaching of Mathematics 2005 which is to meet the emerging needs of all categories of students. Motivating the topics from real life situations and other subject areas, greater emphisis has been laid on application of various concepts.

Objectives

The board objectives of teaching Mathematics at senior school stage intend to help the students :

- to acquire knowledge and critical understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles, symbols and mastery of underlying processes and skills.
- to feel the flow of reasons while proving a result or solving a problem.
- to apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method.
- to develop positive attitude to think, analyze and articulate logically.
- to develop interest in the subject by participating in related competitions.
- to acquaint students with different aspects of Mathematics used in daily life.
- to develop an interest in students to study Mathematics as a discipline.
- to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases.
- to develop reverence and respect towards great Mathematicians for their contirbutions to the field of Mathematics.

COURSE STRUCTURE

CLASS XI (20 .-23)

One Paper Total Period–240 [35 Minutes each]

Three Hours **Max Marks: 80**

No.	Units	No. of Periods	Marks
I.	Sets and Functions	60	23
II.	Algebra	50	25
III.	Coordinate Geometry	50	12
IV.	Calculus	40	08
V.	Statistics and Probability	40	12
	Total	240	80
	Internal Assessment		20

*No chapter/unit-wise weightage. Care to be taken to cover all the chapters.

Unit-I: Sets and Functions

1. **Sets** **(20) Periods**

Sets and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers especially intervals (with notations). Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement.

2. **Relations & Functions** **(20) Periods**

Ordered pairs. Cartesian product of sets. Number of elements in the Cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto R x R x R).Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special type of relation. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum, exponential, logarithmic and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions** **(20) Periods**

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity sin2x + cos2x = 1, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing *sin (x±y)* and *cos (x±y)* in terms of sin*x*, sin*y*, cos*x* & cos*y* and their simple applications. Deducing identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

$$\sin\alpha \pm \sin\beta = 2\sin\frac{1}{2}(\alpha \pm \beta)\cos\frac{1}{2}(\alpha \mp \beta)$$

$$\cos\alpha + \cos\beta = 2\cos\frac{1}{2}(\alpha + \beta)\cos\frac{1}{2}(\alpha - \beta)$$

$$cos\alpha - cos\beta = -2sin\frac{1}{2}(\alpha + \beta)sin\frac{1}{2}(\alpha - \beta)$$

Identities related to sin2*x*, cos2*x*, tan2 *x*, sin3*x*, cos3*x* and tan3*x*.

Unit-II: Algebra

1. **Complex Numbers and Quadratic Equations** **(10) Periods**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane

2. **Linear Inequalities** **(10) Periods**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line.

3. **Permutations and Combinations** **(10) Periods**

Fundamental principle of counting. Factorial *n.* (n!) Permutations and combinations, derivation of Formulae for ^nP_r and ^nC_r and their connections, simple applications.

4. **Binomial Theorem** **(10) Periods**

Historical perspective, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, simple applications.

5. **Sequence and Series** **(10) Periods**

Sequence and Series. Arithmetic Mean (A.M.) Geometric Progression (G.P.), general term of a G.P., sum of *n* terms of a G.P., infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M.

Unit-III: Coordinate Geometry

1. Straight Lines (15) Periods

Brief recall of two dimensional geometry from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axis, point -slope form, slope-intercept form, two-point form, intercept form, Distance of a point from a line.

2. Conic Sections (25) Periods

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-dimensional Geometry (10) Periods

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points.

Unit-IV: Calculus

1. Limits and Derivatives (40) Periods

Derivative introduced as rate of change both as that of distance function and geometrically. Intuitive idea of limit. Limits of polynomials and rational functions trigonometric, exponential and logarithmic functions. Definition of derivative relate it to scope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V Statistics and Probability

1. Statistics (20) Periods

Measures of Dispersion: Range, Mean deviation, variance and standard deviation of ungrouped/grouped data.

2. Probability (20) Periods

Events; occurrence of events, ‘not’, ‘and’ and ‘or’ events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with other theories of earlier classes. Probability of an event, probability of ‘not’, ‘and’ and ‘or’ events.

MATHEMATICS

QUESTION PAPER DESIGN

CLASS – XI (2022-23)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weight age
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	44	55
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	20	25
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	16	20
	Total	80	100

1. *No chapter wise weightage. Care to be taken to cover all the chapters*
2. *Suitable internal variations may be made for generating various templates keeping the overall weightage to different form of questions and typology of questions same.*

Choice(s):

There will be no overall choice in the question paper.

However, 33% internal choices will be given in all the sections

INTERNAL ASSESSMENT	**20 MARKS**
Periodic Tests (Best 2 out of 3 tests conducted)	10 Marks
Mathematics Activities	10 Marks

Note: Please refer the guidelines given under XII Mathematics Syllabus:

CONTENTS

CHAPTER - 1

SETS AND FUNCTIONS

KEY POINTS

- Definition of Set : Set is well defined collection of objects.
- Objects in Set are called elements of Set.
- Elements are said to be 'belong to' set.

  Example: A = {a, b, c, d} is a Set and a, b, c, d are element of Set A

  Here a, b, c, d belongs to A or a, b, c, d $\in$ A

- Representation of Sets:
  - (a) Roster or Tabular form

    e.g.: Set Natural Numbers less than 5 = {1, 2, 3, 4}

  - (b) Set-builder form

    e.g.: Set of Natural Numbers less than 5 = $\{x : x \in N, x < 5\}$

- **Types of sets:**
  - (a) Empty /Null/Void Set: Set which does not contain any element. It is denotd by ϕ or { }
  - (b) Finite set : Set having finite number of elements
  - (c) Infinite set: Set having infinite number of elements
  - (d) Singleton set : Set having only one element
- Cardinal number of finite set: Number of distinct elements of set. It is denoted by n(A).
- Equivalent sets: Two or more finite sets having same number of elements or same cardinal number.

- Subset: A set A is said to be subset of a set B iff $a \in A \Rightarrow a \in B$. $\forall a \in A$

  We write it as $A \subseteq B$.

  Note: ϕ and A itself are always subsets of set A.
- Super set: If $A \subseteq B$ then B is superset of A.
- Proper subset : If $A \subseteq B$, but $A \neq B$ then A is proper subset of B.

  We write it as $A \subset B$.
- Number of subsets of a set A= $2^{n(A)}$
- Number of proper subsets of a Set A = $2^{n(A)} - 1$
- **Equal sets:** Two or more sets having exactly same elements.

  $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

- **Power set:** The collection of all subsets of a set A. It is denoted by P(A)

  $P(A) = \{X : X \subseteq A\}$

  $n[P(A)] = 2^{n(A)}$

- **Types of Intervals**

  (a) Open Interval: $(a, b) = \{ x \in R : a < x < b \}$

  (b) Closed Interval: $[a, b] = \{ x \in R : a \leq x \leq b \}$

  (c) Semi open or Semi closed Interval,

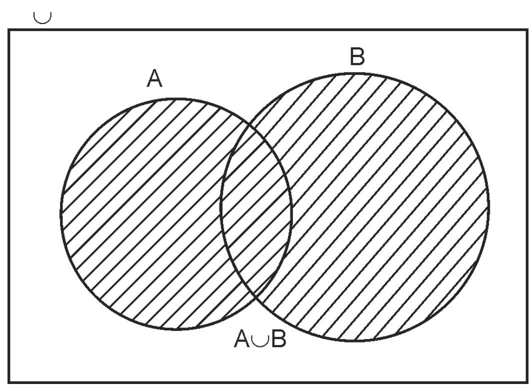
  $(a,b] = \{ x \in R : a < x \leq b\}$

  $[a,b) = \{ x \in R : a \leq x < b\}$

- Venn diagram and operations on sets

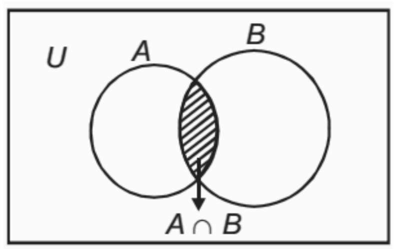
  (a) Union of two sets A and B :

  $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

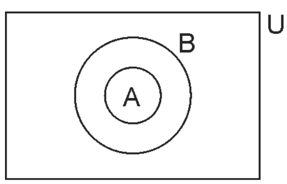


(b) Intersection of two sets A and B :

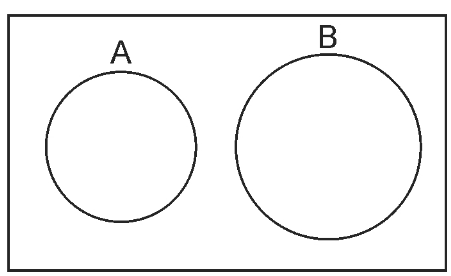
$A \cap B = \{ x : x \in A \text{ and } x \in B\}$



- Subset and superset: $A \subset B$

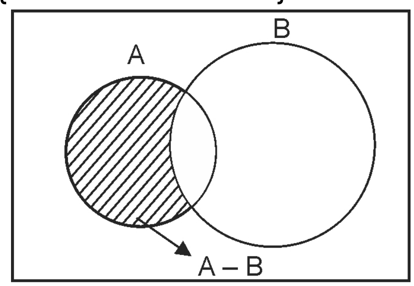


- Disjoint sets: Two sets A and B are said to be disjoint if $A \cap B = \phi$



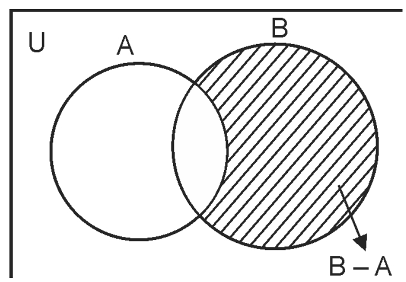
(c) Difference of sets A and B is,

$A - B = \{ x : x \in A \text{ and } x \notin B\}$



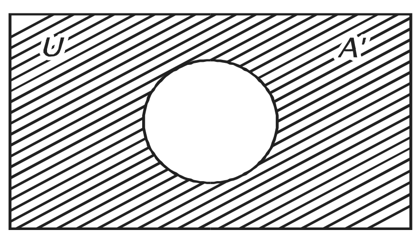
(d) Difference of sets B and A is,

$B - A = \{ x : x \in B \text{ and } x \notin A\}$



(e) Complement of a set A, denoted by A' or A^c

$A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



- Properties of complement of sets :

1. Complement laws

   (i)$A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

   (i)$(A \cup B)' = A' \cap B'$(ii) $(A \cap B)' = A' \cup B'$

   Note **:**This law can be extended to any number of sets.

3. $\phi' = \cup$ and $\cup' = \phi$

4. If $A \subset B$ then $B' \subset A'$

- Laws of Algebra of sets

  (i) $A \cup \phi = A$

  (ii) $A \cap \phi = \phi$

- $A - B = A \cap B' = A - (A \cap B)$
- Commutative Laws :–

  (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$

- Associative Laws :–

  (i) $(A \cup B) \cup C = A \cup (B \cup C)$

  (ii) $(A \cap B) \cap C = A \cap (B \cap C)$

- Distributive Laws :–

  (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$
- $n(A \cup B) + (A \cap B) = n(A) + n(B)$
- If A and B are disjoint, then $n(A \cup B) = n(A) + n(B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

VERY SHORT ANSWER TYPE QUESTIONS

Which of the following are sets? Justify your answer.

1. Write set $\left\{\frac{1}{2},\frac{2}{3},\frac{3}{4},\ldots\frac{19}{20}\right\}$ in set builder form.

2. Write the set $\{x : x \in z^+, x^2 < 4\}$ is Roster form.

   Let A = {1,3,5,7,9}. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces: – (Question- 3,4)

3. (i) 2 _____ A (ii) {3} _____ A (iii) {3, 5} _____ A

4. Write the set A = { x : x is an integer, $-1 \leq x < 4$} in roster form

5. Write the set B = {3,9,27,81} in set-builder form.

   Which of the following are empty sets? Justify. (Question- 6,7)

6. A = { x : x $\in$ N and 3 <x <4}

7. B = { x : x $\in$N and $x^2 = x$}

   Which of the following sets are finite or infinite? Justify. (Question-8, 9)

8. The set of all the points on the circumference of a circle.

9. B = { x : x $\in$ N and x is an even prime number}

10. Are sets A = { –2,2}, B = { x : x $\in$ Z, $x^2 - 4 = 0$} equal? Why?

11. Write (–5,9] in set-builder form

12. Write $\{x : x \in R, -3 \leq x < 7\}$ as interval.

13. If A = { 1,3,5}, how many elements has P(A)?

14. Write all the possible subsets of A = {5,6}.

    If A = Set of letters of the word ‘DELHI’ and B= the set of letters the words ‘DOLL’ find (Question- 15, 16, 17)

15. $A \cup B$

16. $A \cap B$

17. $A - B$

18. Describe the following sets in Roster form

    (i) The set of all letters in the word 'ARITHMETIC'.

    (ii) The set of all vowels in the word 'EQUATION'.

19. Write the set $A = \{x : x \in z, x^2 < 25\}$ in roster form.

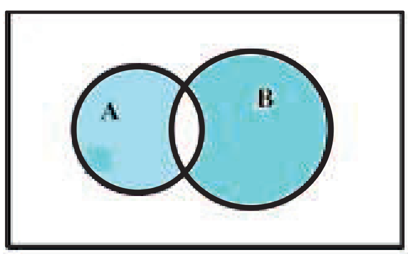
20. Write the set B = {x : x is a two digit numbers, such that the sum of its digits is 7}

SHORT ANSWER TYPE QUESTIONS

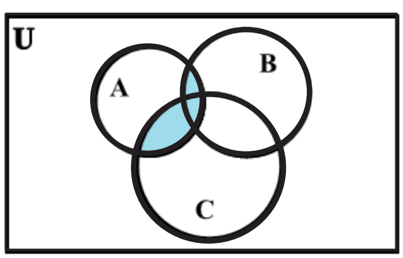
21. Are sets $A = \{1,2,3,4\}$, $B = \{ x : x \in N \text{ and } 5 \leq x \leq 7\}$ disjoint? Justify?

What is represented by the shaded regions in each of the following Venn-diagrams? (Question 22, 23)

22.



23.



SHORT ANSWER TYPE QUESTIONS

24. If A = { 1, 3, 5, 7, 11, 13, 15, 17}

    B = { 2, 4, 6, 8 ... 18}

    And U is universal set then find $A' \cup [(A \cup B) \cap B'$

25. Two sets A and B are such that

    $n(A \cup B) = 21 n(A) = 10 n(B) = 15$ find $n(A \cap B)$ and $n(A - B)$

26. Let A = {1, 2, 4, 5} B = {2, 3, 5, 6} C = {4, 5, 6, 7} Verify the following identity

    $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

27. If $\bigcup$ = { x : x $\in$ N and x ≤ 10}

    A = { x : x is prime and x ≤ 10}
    B = { x : x is a factor of 24}

    Verify the following result

    (i) $A - B = A \cap B'$ (ii) $(A \cup B)' = A' \cap B'$ (iii) $(A \cap B)' = A' \cup B'$

28. For any sets A and B show that

    (i) $(A \cap B) \cup (A - B) = A$ (ii) $A \cup (B - A) = A \cup B$

29. On the Real axis, if A=[0,3]and B=[2,6], then find the following

    (i) A' (ii) $A \cup B$ (iii) $A \cap B$ (iv) $A - B$

30. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

31. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

32. Two sets A and B are such that $n(A \cup B)=21$, $n(A' \cap B')=9$, $n(A \cap B)=7$ find $n(A \cap B)'$.

LONG ANSWER TYPE QUESTIONS

33. In a group of 84 persons, each plays at least one game out of three viz. tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis. What is the importance of sports in daily life?

34. Using properties of sets and their complements prove that

    (i) $(A \cup B) \cap (A \cup B') = A$

    (ii) $A - (A \cap B) = A - B$

    (iii) $(A \cup B) - C = (A - C) \cup (B - C)$

    (iv) $A - (B \cup C) = (A - B) \cap (A - C)$

    (v) $A \cap (B - C) = (A \cap B) - (A \cap C)$.

35. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.

36. A survey shows that 63% people watch news channel A whereas 76% people watch news channel B. If x% of people watch both news channels, then prove that $39 \le x \le 63$.

37. From 50 students taking examination in Mathematics, Physics and chemistry, each of the students has passed in at least one of the subject, 37 passes Mathematics, 24 Physics and 43Chemistry. At most 19 passed Mathematics and Physics, almost 29 Mathematics and chemistry and at most 20 Physics and chemistry. What is the largest possible number that could have passes in all the three subjects?

CASE STUDY TYPE QUESTIONS

38. In a survey of of 600 students of class XI, 150 are using YouTube videos and225 are consulting books (other than text book) as a learning resource. 100were using both YouTube videos and books as a learning resource.

    i. How many students are using either books or YouTube videos as thelearning resource?

    (a) 325 (b) 225 (c) 275 (d) 375

    ii. How many students are neither using YouTube videos nor books asthe learning resource?

    (a) 350 (b) 325 (c) 225 (d) 250

    iii. How many students are using YouTube videos only as the learning resource?

    (a) 50 (b) 100 (c) 150 (d) 125

    iv. How many students are using books only as the learning resource?

    (a) 50 (b) 100 (c) 150 (d) 125

    v. What can be the maximum number of students who will use YouTube video or books as learning resources?

    (a) 600 (b) 375 (c) 275 (d) 500

39. In a class 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics.Of these 13 took both Chemistry and Mathematics, 12 took both physics and chemistry and 11 took both Physics and mathematics. If 6 students were offered all the three subjects Find:

    i. The total number of students are

    (a) 47 (b) 37 (c) 35 (d) 49

ii. How many took Mathematics but not Chemistry?

(a) 11 (b) 1 (c) 6 (d) 12

iii. How many took exactly one of the three subjects?

(a)12 (b) 11 (c) 13 (d) 1

iv. How many took exactly two of these subjects?

(a) 11 (b) 13 (c) 12 (d) 18

v. Number of students who took Physics or Mathematics but notChemistry:

(a)12 (b) 13 (c) 11 (d) 18

40. In a town of 10,000 families, it was found that 40% families go to shop A for their home needs groceries, 20% families go to the shop B and 10% families go to shop C. 5% families go to shops A and B, 3% go to B and C and 4% families go to A and C. 2% families go to all the three shops A, Band C. Find:

i. The number of families which go to shop A only;

(a) 4000 (b) 3300 (c) 3700 (d) 4200

ii. The number of families which don't visit/purchase from any of A, Band C.

(a) 4000 (b) 7000 (c) 3300 (d) 6000

iii. The number of families which don't visit/purchase from any of A, B and C.

(a) 300 (b) 200 (c) 100 (d) 600

iv. The number of families that purchase from exactly one shop.

(a) 4700 (b) 4000 (c) 5200 (d) 3800

v. The number of families that buy from at least one of the shops A, B or C.

(a) 4000 (b) 6000 (c) 7000 (d) 1000

Multiple Choice Questions

41. In set builder method the null set is represented by

(a) { } (b) ϕ (c) $\{ x : x \neq x\}$ (d) $\{ x : x = x\}$.

42. If A and B are two given sets, then $A \cap (A \cap B)'$ is equal to

(a) A (b) B′ (c) ϕ (d) A – B.

43. If A and B are two sets such that $A \subset B$ then $A \cap B'$ is

(a) A (b) B′ (c) ϕ (d) $A \cap B$.

44. If $n(A \cup B) = 18$, $n(A - B) = 5$, $n(B - A) = 3$ then $n(A \cap B)$ is

(a) 18 (b) 10 (c) 15 (d) 12

45. For any two sets A and B, $A \cap (A \cup B)'$ is equal to

(a) A (b) B (c) ϕ (d) $A \cap B$

46. If n(A) = 5 and n(B) = 7, then maximum number of elements in $A \cup B$ is

(a) 7 (b) 5 (c) 12 (d) None of these

47. $n[P\{P(\phi)\}] =$

(a) 2 (b) 4 (c) 8 (d) 0

48. If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is

(a) 120 (b) 30 (c) 31 (d) 32

49. For any two sets A and B, $(A - B) \cap (B - A) =$

(a) $(A - B) \cup A$ (b) $(B - A) \cup B$

(c) $(A \cup B) - (A \cap B)$ (d) $(A \cup B) \cap (A \cap B)$

50. If X = $\{8^n - 7n - 1 : n \in N)$ and $y = \{49n - 49 : n \in N\}$, then

(a) X $\subset$ Y (b) Y $\subset$ X

(d) X = Y (d) X $\cap$ Y = ϕ

ANSWERS

1. $\left\{x : x = \dfrac{n}{n+1}, n \in N, n \leq 19\right\}$
2. {1}
3. (i) $\notin$ (ii) $\notin$ (iii) $\notin$
4. A = {–1, 0, 1, 2, 3}
5. B = { x : x = 3^n, n $\in$ N and 1 $\leq$ n $\leq$ 4}
6. Empty set because no natural number is lying between 3 and 4
7. Non-empty set because B = {1}
8. Infinite set because circle is a collection of infinite points whose distances from the centre is constant called radius.
9. Finite set because B = {2}
10. Yes, because $x^2 - 4 = 0$;x = 2, – 2 both are integers
11. {x : x $\in$ R, –5 < x ≤ 9}
12. [–3,7)
13. $2^3 = 8$
14. ϕ, {5}, {6}, {5,6}
15. A $\cup$ B = {D, E, L, H, I, O}

16. $A \cap B = \{D, L\}$

17. $A - B = \{E, H, I\}$

18. (i) {A, R, I, T, H, M, E, C}   (ii)   {E, U, A, I, O}

19. $\{ -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

20. {16, 25, 34, 43, 52, 61, 70}

21. Yes, because $A \cap B = \phi$

22. $(A - B) \cup (B - A)$ or $A \Delta B$

23. $A \cap (B \cup C)$

24. $\cup$

25. $n(A \cap B) = 4$, $n(A - B) = 6$

29. (i) $(-\infty, 0) \cup (3, \infty)$ (ii) $[0, 6]$ (iii) $[2, 3]$ (iv) $[0, 2)$

30. **Hint:** $\cup$ = set of people surveyed
A = set of people who play cricket
B = set of people who play tennis

Number of people who play neither cricket nor tennis

$$= n[(A \cup B)'] = n(U) - n(A \cup B)$$
$$= 450 - 200$$
$$= 250$$

31. There are 280 students in the group.

32. 23

33. 6

35. n = 3   m = 6

37. 14

38. i. (c)   ii. (b)   iii. (a)   iv. (d)   v. (b)

39. i. (c) ii. (a) iii. (b) iv. (d) v. (a)

40. i. (b) ii. (a) iii. (d) iv. (c) v. (b)

41. (c)

42. (b)

43. (c)

44. (b)

45. (c)

46. (c)

47. (a)

48. (c)

49. (c)

50. (a)

CHAPTER – 2

RELATIONS AND FUNCTIONS

CONCEPT MAP

- **Ordered Pair:** An ordered pair consists of two objects or elements in a given fixed order.

  Remarks: An ordered pair is not a set consisting of two elements. The ordering of two elements in an ordered pair is important and the two elements need not be distinct.

- **Equality of Ordered Pair:** Two ordered pairs (x_1, y_1) & (x_2, y_2) are equal if $x_1 = x_2$ and $y_1 = y_2$.

  i.e. $(x_1, y_1) = (x_2, y_2) \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$

- **Cartesian product of two sets:** Cartesian product of two non-empty sets A and B is given by A × B and $A \times B = \{(x, y) : x \in A$ and $y \in B\}$.

- **Cartesian product of three sets:** Let A, B and C be three sets, then A × B × C is the set of all ordered triplet having first element from set A, 2nd element from set B and 3rd element from set C.

  i.e., $A \times B \times C = \{(x, y, z) : x \in A, y \in B \text{ and } z \in C\}$.

- **Number of elements in the Cartesian product of two sets:** If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

- **Relation:** Let A and B be two non-empty sets. Then a relation from set A to set B is a subset of A × B.

- **No. of relations:** If n(A) = p, n(B) = q then no. of relations from set A to set B is given by 2^{pq}.
- **Domain of a relation:** Domain of R = $\{a : (a,b) \in R\}$
- **Range of a relation:** Range of R = $\{ b : (a,b) \in R\}$
- Co-domain of R from set A to set B = set B.
- Range $\subseteq$ Co-domain
- **Relation on a set:** Let A be non-empty set. Then a relation from A to A itself. i.e., a subset of A × A, is called a relation on a set.
- **Inverse of a relation:** Let A, B be two sets and Let R be a relations from set A to set B.

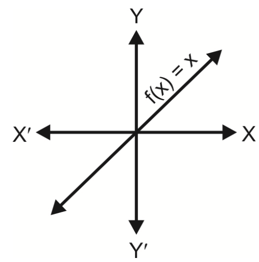
  Then the inverse of R denoted R^{-1} is a relation from set B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$
- **Function:** Let A and B be two non-empty sets. A relation from set A to set B is called a function (or a mapping or a map) if each element of set A has a unique image in set B.

  Remark: If $(a, b) \in f$ then 'b' is called the image of 'a' under f and 'a' is called pre-image of 'b'.
- **Domain and range of a function:** If a function 'f' is expressed as the set of ordered pairs, the domain of 'f' is the set of all the first components of members of f and range of 'f' is the set of second components of member of 'f'.

  i.e., $D_f = \{a : (a, b) \in f\}$ and $R_f = \{b : (a, b) \in f\}$
- **No. of functions:** Let A and B be two non-empty finite sets such that n(A) = p and n(B) = q then number of functions from A to B = q^p.
- **Real valued function:** A function $f : A \to B$ is called a real valued function if B is a subset of R (real numbers).

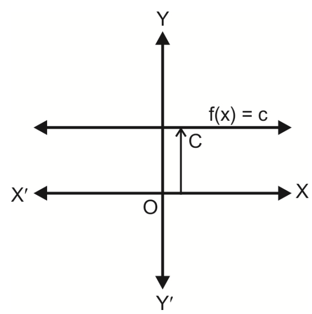
- **Identity function:** f : R→R given by f(x) = x ∀ x ∈ R (real number)

  Here, D_f = R and R_f = R



- **Constant function:** f : R → R given by f(x) = c for all x ∈ R where c is any constant

  Here, D_f = R and R_f = {c}

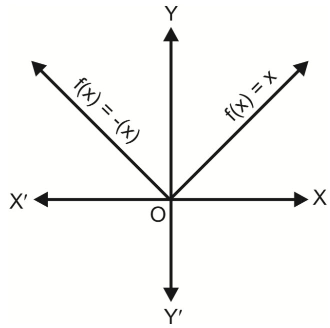


- **Modulus function:** f : R → R given by f(x) = |x| ∀ x ∈ R

  Here, D_f = R and R_f = [0, ∞)

  Remarks : $\sqrt{x^2} = |x|$

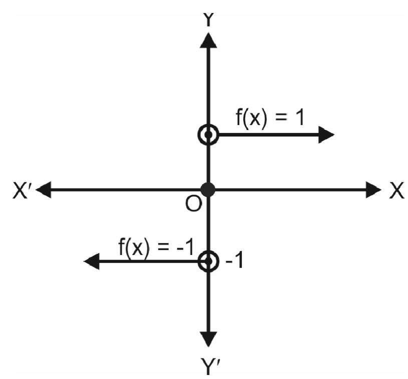
  or $f(x) = |x| = \begin{bmatrix} x : x \geq 0 \\ -x : x < 0 \end{bmatrix}$



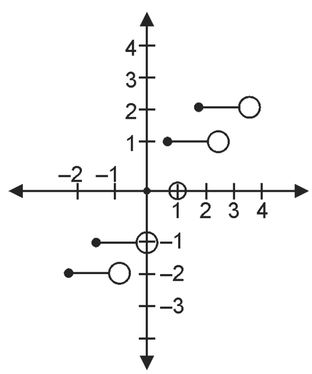
- **Signum function:** f : R → R defined by $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Or

$$f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$

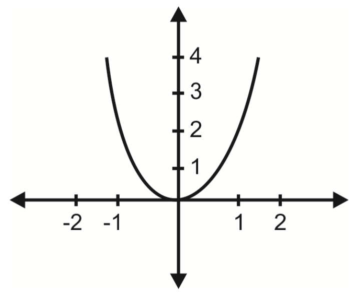


- **Greatest Integer function:** f : R → R defined by f(x) = [x], $x \in R$ assumes the value of the greatest integer, less than or equal to x. Here, $D_f = R$ and $R_f = Z$



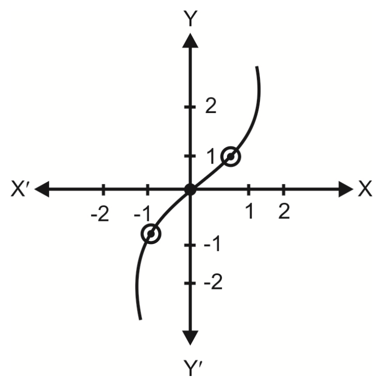
- Graph for f : R → R, defined by $f(x) = x^2$

  Here, $D_f = R$ and $R_f = [0, \infty)$

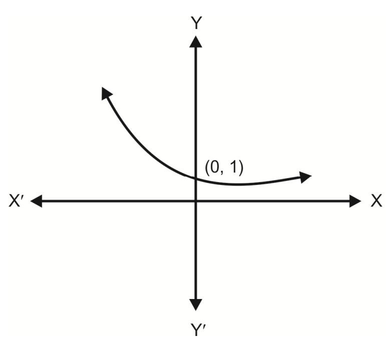


- Graph for f : R → R, defined by $f(x) = x^3$

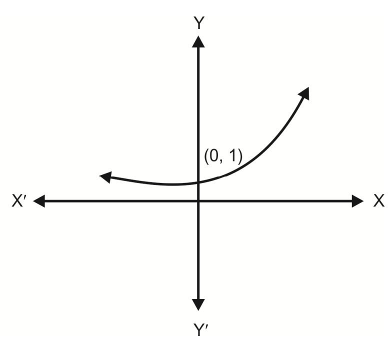
  Here $D_f = R$ and $R_f = R$



- **Exponential function:** f : R → R, defined by $f(x) = a^x$, $a > 0$, $a \neq 1$



When $0 < a < 1$



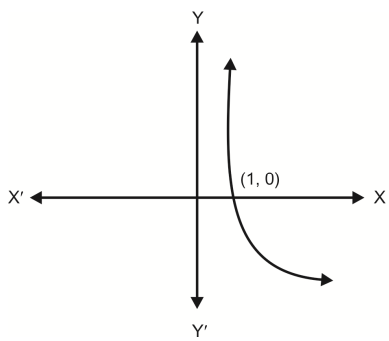
When $a > 1$

$$f(x) = a^x \begin{cases} > 1 & \text{for } x < 0 \\ = 1 & \text{for } x = 0 \\ < 1 & \text{for } x > 0 \end{cases} \qquad f(x) = a^x \begin{cases} < 1 & \text{for } x < 0 \\ = 1 & \text{for } x = 0 \\ > 1 & \text{for } x > 0 \end{cases}$$

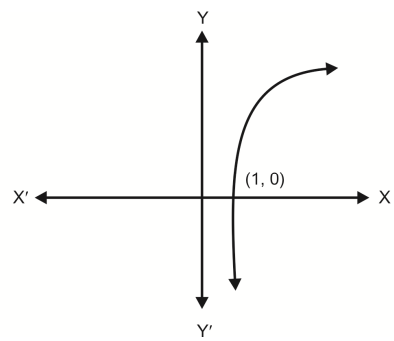
- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots \infty, \quad 2 < e < 3$$

- Logarithmic functions, $f : (0, \infty) \to R$; $f(x) \log_a x$, $a > 0$, $a \neq 1$



$f(x) = \log_a x, \; 0 < a < 1$
$D_f = (0, \infty)$
$R_f = R$
Case I When $0 < a < 1$



$f(x) = \log_a x$, for $a > 1$
$D_f = (0, \infty)$
$R_f = R$
Case II When $a > 1$

- **Natural logarithm function:** $f(x) = \log_e x$ or $\ln(x)$.

- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $x \subset R$ then

$(f \pm g)(x) = f(x) \pm g(x) \; \forall \; x \in X$

$(fg)(x) = f(x)\, g(x) \; \forall \; x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \forall \; x \in X \text{ provided } g(x) \neq 0$$

Very Short Answer Type Question

1. Find a and b if $(a - 1, b + 5) = (2, 3)$

   If $A = \{1,3,5\}$, $B = \{2,3\}$, find : (Question- 2, 3)

2. $A \times B$

3. $B \times A$

   Let $A = \{1,2\}$, $B = \{2,3,4\}$, $C = \{4,5\}$, find (Question- 4, 5)

4. $A \times (B \cap C)$

5. $A \times (B \cup C)$

6. If $P = \{1,3\}$, $Q = \{2,3,5\}$, find the number of relations from P to Q

7. If $R = \{(x,y): x,y \in Z, x^2 + y^2 = 64\}$, then,

   Write R in roster form

   Which of the following relations are functions? Give reason. (Questions 18 to 20)

8. $R = \{ (1,1), (2,2), (3,3), (4,4), (4,5)\}$

9. $R = \{ (2,1), (2,2), (2,3), (2,4)\}$

10. $R = \{ (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)\}$

SHORT ANSWER TYPE QUESTIONS

11. If A and B are finite sets such that $n(A) = 5$ and $n(B) = 7$, then find the number of functions from A to B.

12. If $f(x) = x^2-3x + 1$ find $x \in R$ such that $f(2x) = f(x)$

Let f and g be two real valued functions, defined by, $f(x) = x$, $g(x) = |x|$. Find: (Question 13 to 16)

13. $f + g$

14. $f - g$

15. fg

16. $\dfrac{f}{g}$

17. If $f(x) = x^3$, find the value of, $\dfrac{f(5)-f(1)}{5-1}$

18. Find the domain of the real function, $f(x)=\sqrt{x^2-4}$

19. Find the domain of the function, $f(x)=\dfrac{x^2+2x+3}{x^2-5x+6}$

Find the range of the following functions. (Question- 20, 21)

20. $f(x)=\dfrac{1}{4-x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,
$R = \{(x, y): x, y \in Z, xy = 4\}$

Find the range of the following relations: (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R=\left\{\left(x,\dfrac{1}{x}\right): x \in Z, 0 < x < 6\right\}$

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y): x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.
(b) Find domain of R.
(c) Find range of R.
(d) Write co-domain of R.

26. If A = {2,4,6,9} B = {4,6,18,27,54} and a relation R from A to B is defined by $R = \{(a,b): a \in A, b \in B$, a is a factor of b and $a < b\}$, then find in Roster form. Also find its domain and range.

27. Find the domain and range of,

$f(x) = |2x - 3| - 3$

28. Draw the graph of the Constant functionf : $R \to R$; $f(x) = 2 \ \forall \ x \in R$. Also find its domain and range.

29. Draw the graph of the function $|x - 2|$

Find the domain and range of the following real functions (Question 30-35)

30. $f(x) = \sqrt{x^2 + 4}$

31. $f(x) = \dfrac{x+1}{x-2}$

32. $f(x) = \dfrac{|x+1|}{x+1}$

33. $f(x) = \dfrac{x^2 - 9}{x - 3}$

34. $f(x) = 1 - |x - 3|$

35. $f(x) = \dfrac{1}{\sqrt{9 - x^2}}$

36. Determine a quadratic function (f) is defined by f(x) = ax² + bx +c. If f(0) = 6; f(2) = 11, f(–3) = 6

37. Draw the graph of the function $f(x) = \begin{cases} 1+2x & x<0 \\ 3+5x & x \geq 0 \end{cases}$ also find its range.

38. Draw the graph of following function

$$f(x) = \begin{cases} \dfrac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Also find its range.

Find the domain of the following function.

39. $f(x) = \dfrac{1}{\sqrt{x + |x|}}$

40. $f(x) = \dfrac{1}{\sqrt{x - |x|}}$

41. $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

42. $f(x) = \sqrt{4 - x} + \dfrac{1}{\sqrt{x^2 - 1}}$

43. Find the domain for which the following functions:
$f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

44. If $f(x) = x - \dfrac{1}{x}$ prove that $\left[f(x)\right]^3 = f(x^3) + 3f\left(\dfrac{1}{x}\right)$.

45. If [x] denotes the greatest integer function. Find the solution set of equation, $[x]^2 + 5[x] + 6 = 0$.

46. If $f(x) = \dfrac{ax - b}{bx - a} = y$. Find the value of f(y).

Long Answer Type Questions

47. Draw the graph of following function and find range (R_f) of

$f(x) = |x - 2| + |2 + x| \quad \forall \quad -3 \le x \le 3.$

48. Find domain and range $f(x) = \dfrac{1}{2 - \sin 3x}$

CASE STUDY TYPE QUESTIONS

49. To make himself self-dependent and to earn his living, a person decidedto setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture.

i. If x units of hand sanitizers are manufactured per month. What is thecost function?

(a) 15000 – 30x (b) 15000 + 30x

(c) 15000 + x (d) 15000 + 31x

ii. If each unit is sold for Rs. 45. What is the selling (revenue) function?

(a)30x (b) 45 + x

(c) 45x (d) 45 + 30x

iii. What is the profit function?

(a) 15x + 15000 (b) 15(x– 1000)

(c) 15x (d) None of these

iv. For Break even (No Profit, no loss situation) in a month, how manyunits should be manufactured and sold?

(a) 500 (b) 750

(c) 1000 (d) 1500

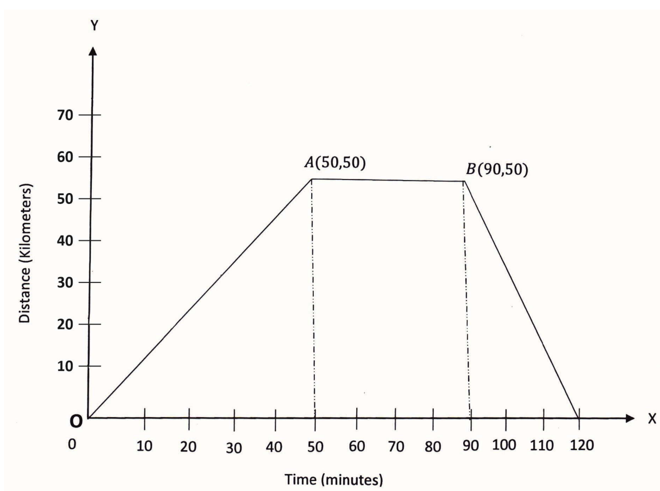
v. What is the monthly cost borne by the person if he decided tomanufacture 1500 units in a month?

(a) 15000 (b) 30000

(c) 45000 (d) 60000

50. This is a graph showing how far the distances have been travelled by Sunita (in her car) in a given time.

She drove, stopped, does her work and returned back.



i. The line OA of the graph represents the function:

(a) $x=10y$ (b) $y = 10x$

(c) $y=x$ (d) $y=\frac{x}{2}$

ii. At what distance from the starting point Sunita stopped to do her work?

(a) 30m (b) 40m

(c) 50m (d) 60m

iii. How much time Sunita took to complete her work?

(a) 30 min (b) 40 min

(c) 50 min (d) 60 min

iv. Line AB represents the constant function:

(a) $y = 50$ (b) $x = 50$

(c) $y = 10$ (d) $x = 9$

v. How much time Sunita took to reach at a distance of 40 km. from the initial point?

(a) 30 min (b) 40 min

(c) 50 min (d) 1 hour

Multiple Choice Questions

51. If A = {1, 2, 4}, B = {2, 4, 5}, C = {2, 5} then (A – B) × (B – C)

(a) {(1, 2), (1, 5), (2, 5)} (b) {1, 4}

(c) {1, 4} (d) None of these.

52. If R is a relation on set A = {1, 2, 3, 4, 5, 6, 7, 8}

given by xRy ⇔ y = 3x, then R = ?

(a) {(3, 1), (6, 2), (8, 2), (9, 3)} (b) {(3, 1), (6, 2), (9, 3)}

(c) {(3, 1), (2, 6), (3, 9)} (d) None of these.

53. Let A = {1, 2, 3}, B = {4, 6, 9} if relation R from A to B defined by x is greater then y. the range of R is -

(a) {1, 4, 6, 9} (b) {4, 6, 9}

(c) {1} (d) None of these.

54. If R be a relation from a set A to a set B then -

(a) R = A ∪ B (b) R = A ∩ B

(c) R ⊆ A × B (d) R ⊆B × A.

55. If $2f(x) - 3f\left(\frac{1}{x}\right) = x^2$ ($x \neq 0$), then f(2) is equal to -

(a) $\frac{-7}{4}$ (b) $\frac{5}{2}$

(c) –1 (d) None of these.

56. Range of the function f(x) = cos[x] for $\frac{-\pi}{2} < x < \frac{\pi}{2}$ is -

(a) {–1, 1, 0} (b) {cos1, cos2, 1}

(c) {cos1, –cos1, 1} (d) {–1, 1}.

57. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x + x^3}{1 + 3x^2}$ then f{g(x)} is equal to -

(a) f(3x) (b) $\{f(x)\}^3$

(c) 3f(x) (d) –(f(x).

58. If f(x) = cos(logx) then value of $f(x).f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\}$ is -

(a) 1 (b) –1

(c) 0 (d) ±1.

59. Doman of $f(x) = \sqrt{4x - x^2}$ is -

(a) R – [0, 4] (b) R – (0, 4)

(c) (0, 4) (d) [0, 4].

60. If $[x]^2 - 5[x] + 6 = 0$, where [.] denote the greatest integer function then -

(a) $x \in [3, 4]$ (b) $x \in (2, 3]$

(c) $x \in [2, 3]$ (d) $x \in [2, 4)$.

ANSWERS

1. $a = 3, b = -2$

2. $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

3. $B \times A = \{ (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$

4. $\{(1,4), (2,4)\}$

5. $\{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\}$

6. $2^6 = 64$

7. $R = \{ (0,8), (0,-8), (8,0), (-8,0)\}$

8. Not a function because 4 has two images.

9. Not a function because 2 does not have a unique image.

10. Function because every element in the domain has its unique image.

11. 7^5

12. 0,1

13. $f + g = \begin{cases} 2x & x \geq 0 \\ 0 & x < 0 \end{cases}$

14. $f - g = \begin{cases} 0 & x \geq 0 \\ 2x & x < 0 \end{cases}$

15. $fg = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

16. $\frac{f}{g} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$ and Note:- $\frac{f}{g}$ is not defined at x = 0

17. 31

18. $(-\infty, -2] \cup [2, \infty)$

19. R – {2,3}

20. $(-\infty, 0) \cup [1/4, \infty)$

21. $[2,\infty)$

22. {–4, –2, –1,1,2,4}

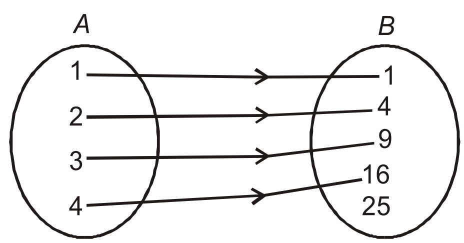
23. {2,4,6,8}

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

23. {2,4,6,8}

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

25. (a)



(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

26. R = { (2,4) (2,6) (2,18) (2,54) (6,18) (6,54) (9,18) (9,27) (9,54) }

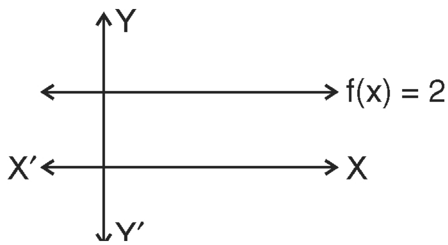
Domain is R = {2,6,9}

Range of R = { 4, 6, 18, 27, 54}

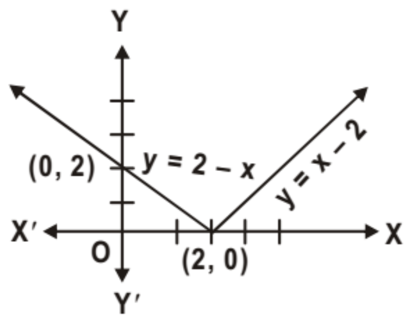
27. Domain is R

Range is $[-3, \infty)$

28. Domain = R,Range = {2}



29.



30. Domain = R,

    Range =[2, ∞)

31. Domain = R – {2}

    Range = R – {1}

32. Domain = R – {–1}

    Range = {1, –1}

33. Domain = R – {3}

    Range = R – {6}

34. Domain = R

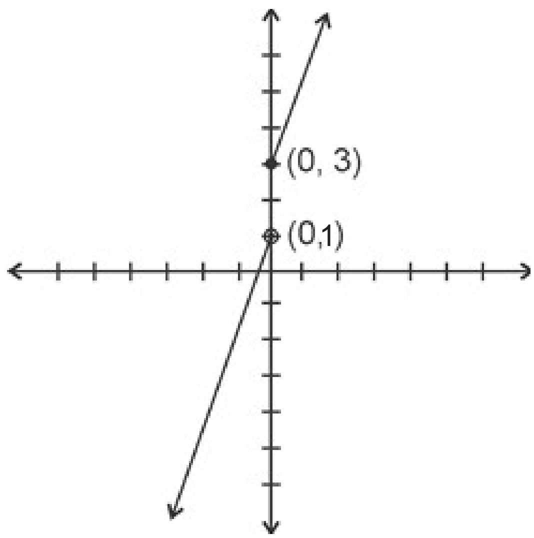
    Range =(–∞, 1]

35. Doman = (–3, 3)

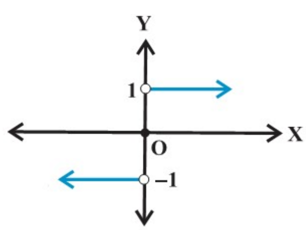
    Range = $\left[\frac{1}{3}, \infty\right)$

36. $\frac{1}{2}x^2+\frac{3}{2}x+6$

37. $(-\infty, 1) \cup [3, \infty)$



38. Range of f = $\{-1,0,1\}$



39. $(0, \infty)$

40. ϕ(given function is not defined)

41. $(-\infty, -2) \cup (4, \infty)$

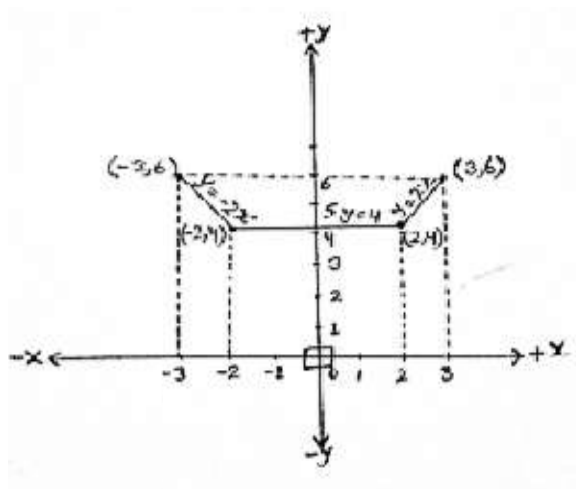
42. $(-\infty, -1) \cup (1, 4]$

43. $\left\{-2, \frac{1}{2}\right\}$

45. $[-3,-1)$

46. x

47. $R_f = [4, 6]$and graph is



48. Domain = R
    Range = [1/3, 1]

49. i. (b) ii. (c) iii. (b) iv. (c) v. (d)

50. i. (c) ii. (c) iii. (b) iv. (a) v. (b)

51. (b)

52. (d)

53. (c)

54. (c)

55. (a)

56. (b)

57. (c)

58. (c)

59. (d)

60. (d)

CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- 1 radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.
- πradian = 180 degree

  $$1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ} = 57^{\circ}\ 16'\ 22'' \text{ (Appr.)}$$

- If an arc of length 'ℓ' makes an angle 'θ' radian at the centre of a circle of radius 'r', we have $\theta = \frac{\ell}{r}$.
- 1 degree is $\left(\frac{1}{360}\right)^{th}$ part of a circle. One degree is further divided into 60 parts called minutes and one minute is further divided into 60 parts called seconds.
- 360° = one complete revolution
- $1^{\circ} = 60'$(minutes)
- $1' = 60''$(second)

Quadrant →	I	II	III	IV
t-functions which are postive	All	sinx cosecx	tanx cotx	cosx secx

Function	Domain	Range
sinx	R	[–1, 1]
cosx	R	[–1, 1]
tanx	$R - \left\{(2n+1)\frac{\pi}{2}\right\};\ n \in z$	R
cosecx	$R - \{n\pi\};\ n \in z$	R – (–1, 1)
secx	$R - \left\{(2n+1)\frac{\pi}{2}\right\};\ n \in z$	R – (–1, 1)
cotx	$R - \{n\pi\};\ n \in z$	R

- **Trigonometric Identities:**

(i) sin (x + y) = sinx cosy + cosx siny

(ii) sin (x – y) = sinx cosy – cosx siny

(iii) cos (x + y) = cosx cosy – sinx siny

(iv) cos (x – y) = cosx cosy + sinx siny

(v) $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x . \tan y}$

(vi) $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x . \tan y}$

(vii) $\cot(x+y) = \dfrac{\cot x . \cot y - 1}{\cot y + \cot x}$

(viii) $\cot(x-y) = \dfrac{\cot x . \cot y + 1}{\cot y - \cot x}$

(ix) $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$

(x) $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

(xi) $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

(xii) $\sin 3x = 3\sin x - 4\sin^3 x$

(xiii) $\cos 3x = 4\cos^3 x - 3\cos x$

(xiv) $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

(xv) $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

(xvi) $\cos x - \cos y = 2\sin\dfrac{x+y}{2}\sin\dfrac{y-x}{2}$

(xvii) $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

(xviii) $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

(xix) $2\sin x \cos y = \sin(x + y) + \sin(x - y)$

(xx) $2\cos x \sin y = \sin(x + y) - \sin(x - y)$

(xxi) $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

(xxii) $2\sin x \sin y = \cos(x - y) - \cos(x + y)$

(xxiii) $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

(xxiv) $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

(xxv) $\tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

sign '+' or '–' will be decided according to the quadrant in which angle lies.

- Maximum and minimum values of the expression Acosθ + Bsinθ are $\sqrt{A^2+B^2}$ and $-\sqrt{A^2+B^2}$ respectively, where A and B are constants.

VERY SHORT ANSWER TYPE QUESTIONS

1. Write the radian measure of 5° 37’ 30”.
2. Write the degree measure of $\frac{11}{16}$ radian.
3. Write the value of $\tan\left(\frac{19\pi}{3}\right)$.
4. What is the value of sin (–1125°).
5. Write the value of 2sin75° sin15°.
6. What is the maximum value of 3 – 7cos5x.
7. Express sin12θ + sin4θ as the product ofsines and cosines.
8. Express 2cos4x sin2x as an algebraic sum of sines and cosines.
9. Write the maximum value of cos(cosx) and also write its minimum value.
10. Write is the value of $\tan\frac{\pi}{12}$.

SHORT ANSWER TYPE QUESTIONS

11. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°.
12. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$ Find cos *A*, sin 2*A*.
13. What is the sign of cos x/2 – sin x/2 when

    (i) $0 < x < \pi/4$ (ii) $\frac{\pi}{2} < x < \pi$

14. Prove that cos 510° cos 330° + sin 390° cos 120° = –1.

15. Find the maximum and minimum value of 7 cos x + 24 sin x.

16. Evaluate sin(π + x) sin(π – x) cosec^2x.

17. Find the angle in radians between the hands sof a clock at 7 : 20 PM.

18. If $\cot\alpha = \frac{1}{2}$ $\sec\beta = \frac{-5}{3}$ where π<α< 3 π /2 and $\frac{\pi}{2} < \beta < \pi$. Find the value of tan (α + β).

19. If $\cos x = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$. Find the value of cos x/2 , tan x/2

20. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$

21. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

22. Find the minimum and maximum value ofsin^4x + cos^2x; $x \in R$

23. Find x if tan (x – 15°) = tan (x + 15°)

24. If $\sec x = \sqrt{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find the value of

$$\frac{1 - \tan x - \operatorname{cosec} x}{1 - \cot x - \operatorname{cosec} x}$$

25. If $f(x) = \frac{\cot x}{1 + \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$ then find $f(\alpha).f(\beta)$.

26. Prove that tan 70° = tan 20° + 2 tan 50°

27. Prove that tan 13x = tan 4x + tan 9x + tan 4x tan 9x tan 13x.

Prove the following Identities

28. $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta . \cos 4\theta$ **.**

29. $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ **.**

30. $\dfrac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x . \sin x + \cos 6x . \cos x} = \tan 2x$ **.**

31. $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$ **.**

32. $\tan\alpha. \ \tan(60° - \alpha). \tan(60° + \alpha) = \tan 3\alpha.$

33. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$ **.**

34. $\dfrac{\cos x}{1 - \sin x} = \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$ **.**

35. $\cos 10° + \cos 110° + \cos 130° = 0.$

36. $\dfrac{\sin(x+y) - 2\sin x + \sin(x-y)}{\cos(x+y) - 2\cos x + \cos(x-y)} = \tan x$

37. $\sin x + \sin 2x + \sin 4x + \sin 5x = 4\cos\dfrac{x}{2} . \cos\dfrac{3x}{2} . \sin 3x$

38. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

39. Find the value of $\sqrt{3}$ cosec 20° – sec 20°

40. Draw the graph of cos$x$, sin$x$ and tan$x$ in [0, 2π].

41. Draw sin$x$, sin2x and sin3x on same graph and with same scale.

42. Evaluate: $\tan\left(\frac{13\pi}{12}\right)$

43. If tan A – tan B = x, cot B – cot A = y prove that cot (A – B) $= \frac{1}{x} + \frac{1}{y}$

44. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$ then prove that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

45. Find the range of 5 sin$x$ – 12 cos$x$ + 7.

46. Show that $\cos^2 + \cos^2\left(x + \frac{2\pi}{3}\right) + \cos^2\left(x - \frac{2\pi}{3}\right) = \frac{3}{2}$

47. Show that $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)$

Long Answer Type Questions

48. Find $\cos {}^{\pi}\!/_{8}$

49. Prove that sin 10° sin 30° sin 50° sin 70° = $\frac{1}{16}$.

50. $\cos\frac{\pi}{5}.\cos\frac{2\pi}{5}.\cos\frac{4\pi}{5}.\cos\frac{8\pi}{5} = \frac{1}{16}$

51. $\cos 20°.\cos 40°.\cos 80° = \frac{1}{8}$

52. Evaluate: $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right)$

53. If $\cos x = \cos\alpha.\cos\beta$ then prove that $\tan\left(\frac{x+\alpha}{2}\right).\tan\left(\frac{x-\alpha}{2}\right) = \tan^2\frac{\beta}{2}$

54. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$ then prove that $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$.

55. If $\sin(\theta+\alpha) = a$ and $\sin(\theta+\beta) = b$ then prove that

$$\cos 2(\alpha-\beta) - 4ab\cos(\alpha-\beta) = 1 - 2a^2 - 2b^2$$

56. If α and β are the solution of the equation, $a\tan\theta + b\sec\theta = c$, then show that $\tan(\alpha+\beta) = \frac{2ac}{a^2 - c^2}$.

57. Prove that

$$\cos^2 x + \cos^2 y - 2\cos x.\cos y.\cos(x+y) = \sin^2(x+y)$$

58. Prove that :

$$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$$

59. Prove that : $\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16.\sin A}$.

60. Evaluate: $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

61. Prove that : $4\sin\alpha.\sin\left(\alpha+\frac{\pi}{3}\right).\sin\left(\alpha+\frac{2\pi}{3}\right) = \sin 3\alpha$.

62. If sin A + sin B = p, cosA + cosB = q show that

(i) $\sin(A+B) = \frac{2pq}{p^2+q^2}$

(ii) $\cos(A+B) = \frac{p^2-q^2}{p^2+q^2}$

(iii) $\tan(A+B) = \frac{p^2+q^2}{p^2-q^2}$

63. Show that $\sin^3 x + \sin^3\left(\frac{2\pi}{3}+x\right) + \sin^3\left(\frac{4\pi}{3}+x\right) = \frac{-3}{4}\sin 3x$

CASE STUDY TYPE QUESTIONS

64. After retirement, Mr. D. N. Sharma purchased a farm house in shape of quadrilateral ABCD with $\angle A = 90°$, $\angle B = 72°$, $\angle C = 108°$ and $\angle D = 90°$. He also purchased a horse and cow. One day, he tied the horse with a rope at vertex B and oserved that it describes an arc of length 88 m when it moves along a circular path keeping the rope tight.

Based on above information answer the following :-

i. What is radian measure of $\angle B$?

(a) $2\pi/5$ (b) $3\pi/5$ (c) $\pi/5$ (d) $3\pi/10$

ii. What is length of rope?

(a) 50 m (b) 60 m (c) 70 m (d) 80 m

iii. What will be the length of arc described by horse if he doubles the rope length?

(a) 44 m (b) 176 m (c) 132 m (d) 156 m

iv. What will be the length of arc described by cow if it is tied at vertex c with the rope of same length as horse?

(a) 156 m (b) 132 m (c) 144 m (d) 176 m

v. What is the ratio of area that horse can cover to that of cow with same length of rope?

(a) 1 : 1 (b) 3 : 2 (c) 2 : 3 (d) 2 : 5

65. While playing with this nephew Shashank, Mr. V.S. Malik observes a vertical pole in park. A wire is tied from top of pole to a point on ground level. Mr. Malik asks Shashank some mathematics related questions. Mr. Shashank is Class-XI student and very intelligent in Maths. Using some tools he measure the distance of point at ground where wire is tied as 10 m. and angle between wire and ground level as 75°.

Based on above information answer the following :-

i. What is the value of tan75° ?

(a) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (c) $\frac{\sqrt{3}}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}}{\sqrt{3}-1}$

ii. What is the height of pole?

(a) $10\left(\sqrt{3}+1\right)$ (b) $10\left(\sqrt{3}-1\right)$

(c) $10\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (d) $10\frac{\sqrt{3}-1}{\sqrt{3}+1}$

iii. What is the value of sin75° ?

(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

iv. What is the length of wire?

(a) $10\sqrt{2}\left(\sqrt{3}+1\right)$ (b) $10\left(\sqrt{3}+1\right)$

(c) $10\sqrt{2}\left(\sqrt{3}-1\right)$ (d) $10\left(\sqrt{3}-1\right)$

iii. What is the value of sin105° ?

(a) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Multiple Choice Questions:

66. The greatest value of sinx cosx is -

(a) 1 (b) 2

(c) $\sqrt{2}$ (d) $\frac{1}{2}$.

67. The value of tan1° × tan2° × tan3° ……..…. tan89° is -

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) Not defined.

68. The value of cos1° × cos2° × cos3° ……..… cos179° is -

(a) $\frac{1}{\sqrt{2}}$ (b) 0

(c) 1 (d) –1.

69. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is -

(a) 1 (b) $\sqrt{3}$

(c) $\frac{\sqrt{3}}{2}$ (d) 2.

70. The value of sin50° – sin70° + sin10° is equal to -

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) 2.

71. If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is equal to -

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) 2.

72. If $\alpha + \beta = \frac{\pi}{4}$, then value of $(1 + \tan\alpha) . (1 + \tan\beta)$ is -

(a) 1 (b) 2

(c) –2 (d) Not defined.

73. If $\cos x = \frac{1}{2}\left(a+\frac{1}{a}\right)$,then cos3x is -

(a) $\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$ (b) $\frac{3}{2}\left(a^3+\frac{1}{a^3}\right)$

(c) $\frac{1}{2}\left(a^3-\frac{1}{a^3}\right)$ (d) $\frac{3}{2}\left(a^3-\frac{1}{a^3}\right)$.

74. If $P = 2\sin^2 x - \cos^2 x$, then P lies in the interval -

(a) [1, 3] (b) [1, 2]

(c) [–1, 2] (d) None of these.

75. If $\frac{\pi}{4} < x < \frac{\pi}{2}$, then write the value of $\sqrt{1-\sin 2x}$ is -

(a) cosx – sinx (b) cosx + sinx

(c) sinx – cosx (d) 2.

ANSWERS

1. $\frac{\pi}{32}$
2. 39° 22’ 30”
3. $\sqrt{3}$
4. $-\frac{1}{\sqrt{2}}$
5. $\frac{1}{2}$
6. 10
7. 2 sin8θ cos4θ
8. $\sin 6x - \sin 2x$
9. 1 and –1
10. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

11. 70m

12. $\dfrac{-4}{5}, \dfrac{-24}{25}$

13. (i) +ve(ii) –ve

15. Max value 25;
Min value –25

16. –1

17. $\dfrac{5\pi}{9}$

18. $\dfrac{2}{11}$

19. $-1/\sqrt{3}, -2$

20. $\pi/4$

21. 70 m

22. $\min = \dfrac{3}{4}, \max = 1$

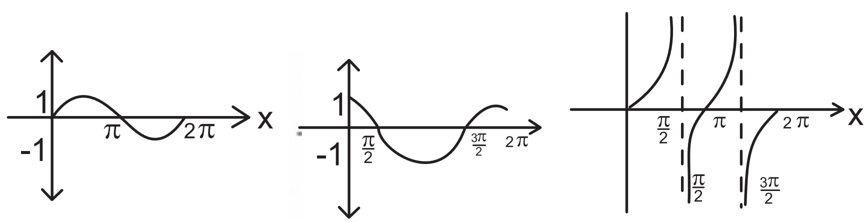
23. 30°

24. 1

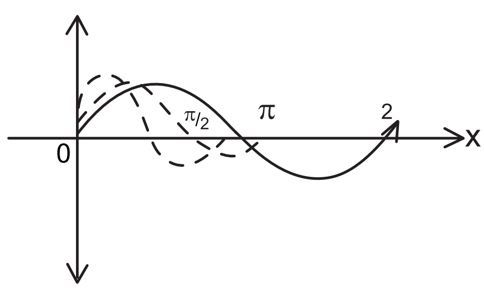
25. $\dfrac{1}{2}$

39. 4

40.



41.



42. (i) $2-\sqrt{3}$

52. $\dfrac{3}{2}$

45. [–6, 20]

48. $= \sqrt{\dfrac{\sqrt{2}+1}{2\sqrt{2}}}$

60. $\dfrac{1}{8}$

64. i. (a) ii. (c) iii. (b) iv. (c) v. (c)

65. i. (b) ii. (c) iii. (c) iv. (a) v. (c)

66. (d)

67. (c)

68. (c)

69. (c)

70. (b)

72. (c)

73. (a)

74. (c)

75. (c)

CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $i^2 = -1$; $i^\circ = 1$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, if atleast one number is positive.
- A number of the form z = a + ib, where a, b $\in$ R is called a complex number.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- a + ib = c + id $\Leftrightarrow$ a = c, and b = d
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$ but if b, d = 0 and a > c then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- 0 + *i* 0 is additive identity of a complex number.
- $-z = -a - ib$ is called the Additive Inverse or negative of z = a + ib

- $1 + i\,0$ is multiplicative identity of complex number.

- $z^{-1} = \frac{1}{z} = \frac{a-ib}{a^2+b^2} = \frac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of

  $z = a + ib$ ($a \neq 0$, $b \neq 0$)

- $\bar{z} = a - ib$ is called conjugate of $z = a + ib$

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane

- $|z_1 + z_2| \leq |z_1| + |z_2|$; $|z_1 - z_2| \geq |z_1| - |z_2|$

- $|z_1 z_2| = |z_1|.\,|z_2|$; $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ provided $|Z_2| \neq 0$

- $\left|z^n\right| = |z|^n$; $|z| = |\bar{z}| = |-z| = |-\bar{z}|$ provided $Z_2 \neq 0$

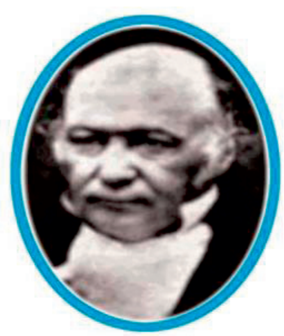
- $\overline{(z_1 \pm z_2)} = \bar{z}_1 \pm \bar{z}_2$; $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

- $\overline{(z^n)} = (\bar{z})^n$

- $z\,.\,\bar{z} = |z|^2$

- For the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in R$, $a \neq 0$, if $b^2 - 4ac < 0$ then it will have complex roots given by,

  $$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$



W. R. Hamilton
(1805-1865)

VERY SHORT ANSWER TYPE QUESTIONS

1. Write the value of $i + i^{10} + i^{20} + i^{30}$

2. Write the additive Inverse of $6i - i\sqrt{-49}$

3. Write the multiplicative Inverse of $1 + 4\sqrt{3}\ i$

4. Write the conjugate of $\dfrac{2-i}{(1-2i)^2}$

5. Write in the form of $a + ib$: $\dfrac{1}{-2+\sqrt{-3}}$

6. Multiply $2 - 3\,i$ by its conjugate.

7. What is the least integral value of K which makes the roots of the equation $x^2 + 5x + k = 0$ imaginary?

8. Find the real value of 'a' for which $3i^3 - 2ai^2 + (1-a)i$ is real.

9. Find the value of $\left(-\sqrt{-1}\right)^{4n-3}$, when n $\in$ N.

10. If a complex number lies in the third quadrant, then find the quadrant of it's conjugate.

11. Find the value of $\sqrt{-25} \times \sqrt{-9}$

12. Evaluate :

    (i) $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$

    (ii) $i\sqrt{-16} + i\sqrt{-25} + \sqrt{49} - i\sqrt{-49} + 14$

    (iii) $\left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3$

(iv) $\dfrac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

13. Find x and y if (x + iy) (2 – 3i) = 4 + i.

14. If n is any positive integer, write value of $\dfrac{i^{4n+1}-i^{4n-1}}{2}$

15. If $z_1 = \sqrt{2}(\cos 30° + i \sin 30°)$, $z_2 = \sqrt{3}(\cos 60 + i \sin 30°)$

    Find $R_e\ (z_1 z_2)$

16. If $|z+4| \leq 3$ then find the greatest and least values of $|z+1|$.

17. Find the real value of a for which $3i^3 - 2ai^2 + (1-a)i + 5$ is real.

SHORT ANSWER TYPE QUESTIONS

18. If x + iy = $\sqrt{\dfrac{1+i}{1-i}}$ prove that $x^2 + y^2 = 1$

19. Find real value of θ such that, $\dfrac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number.

20. If $\left|\dfrac{z-5i}{z+5i}\right| = 1$ show that z is a real number.

21. Find real value of x and y if $\dfrac{(1+i)x-2i}{3+i} + \dfrac{(2-3i)y+i}{3-i} = i$.

22. If $(1+i)(1+2i)(1+3i).......(1+ni) = x+iy$.

    Show, $2.5.10..........(1+n^2) = x^2 + y^2$

23. If z = 2 – 3i show that $z^2 - 4z + 13 = 0$, hence find the value of $4z^3 - 3z^2 + 169$.

24. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = a + ib$, find a and b.

25. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$.

26. If $\left(\frac{2+2i}{2-2i}\right)^n = 1$, find the least positive integral value of n

27. If $(x+iy)^{\frac{1}{3}} = a + ib$ prove $\left(\frac{x}{a} + \frac{y}{b}\right) = 4\left(a^2 - b^2\right)$.

28. Solve

(i) $x^2 - \left(3\sqrt{2} - 2i\right)x - 6\sqrt{2}i = 0$ (ii) $ix^2 - 4x - 4i = 0$

29. Solve $|z + 1| = z + 2(1 + i)$

30. If $|z^2 - 1| = |z|^2 + 1$, then show that z lies on imaginary axis.

31. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represent a circle find its centre and radius.

32. Find all non-zero complex number z satisfying $\bar{z} = iz^2$.

33. If $iz^3 + z^2 - z + i = 0$ then show that $| z | = 1$.

34. If z_1, z_2 are complex numbers such that, $\frac{2z_1}{3z_2}$ is purely imaginary number then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$.

35. If z_1 and z_2 are complex numbers such that,

$\left|1 - \overline{z_1} z_2\right|^2 - \left|z_1 - z_{2^2}\right|^2 = k\left(1 - |z_1|^2\right)\left(1 - |z_2|^2\right)$. Find value of k.

LONG ANSWER TYPE QUESTIONS

36. Find number of solutions of $z^2 + |z|^2 = 0$.

37. If z_1, z_2 are complex numbers such that $\left|\dfrac{z_1 - 3z_2}{3 - z_1.\bar{z}_2}\right| = 1$ *and* $|z_2| \neq 1$ then find $|z_1|$.

38. Evaluate $x^4 - 4x^3 + 4x^2 + 8x + 44$, When $x = 3 + 2i$

39. If $z = x + iy$ and $w = \dfrac{1 - iz}{z - i}$ show that if $|w| = 1$ then z is purely real.

40. If $\left(\dfrac{1+i}{1+2^2 i}\right) \times \left(\dfrac{1+3^2 i}{1+4^2 i}\right) \times .... \times \left(\dfrac{1+(2n-1)^2 i}{1+(2n)^2 i}\right) = \dfrac{a+ib}{c+id}$ then show that $\dfrac{2}{17} \times \dfrac{82}{257} \times .... \times \dfrac{1+(2n-1)^4}{1+(2n)^4} = \dfrac{a^2+b^2}{c^2+d^2}$.

41. Find the values of x and y for which complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate to each other.

42. Show that the complex number z_1, z_2 and z_3 satisfying $\dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{1 - i\sqrt{3}}{2}$ are the vertices of a equilateral triangle.

43. If $f(z) = \dfrac{7 - z}{1 - z^2}$ where $z = 1 + 2i$ then show that $|f(z)| = \dfrac{|z|}{2}$.

44. If z_1, z_2, z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = \left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3}\right| = 1$ then find the value of $|z_1 + z_2 + z_3|$

CASE STUDY TYPE QUESTIONS

45. While solving a typical equation a person finds that one of the root of the equation is a complex number $z = \frac{1+2i}{1-3i}$, help him to find

i. The standard form of z

(a) $-\frac{1}{2}+\frac{i}{2}$ (b) $\frac{1}{2}-\frac{i}{2}$ (c) $-\frac{1}{2}-\frac{i}{2}$ (d) $\frac{1}{2}+\frac{i}{2}$

ii. If $z = 2x+(4-y)i$, then

(a) $x=\frac{1}{4},\ y=\frac{7}{2}$ (b) $x=-\frac{1}{4},\ y=\frac{7}{2}$

(c) $x=\frac{1}{4},\ y=-\frac{7}{2}$ (d) $x=-\frac{1}{4},\ y=-\frac{7}{2}$

iii. Conjugte of Z is

(a) $\frac{1-2i}{1-3i}$ (b) $\frac{1+2i}{1+3i}$ (c) $\frac{1+2i}{1-3i}$ (d) $\frac{1-2i}{1+3i}$

iv. The modulus of z is

(a)1/3 (b) 1/2 (c) $1/\sqrt{2}$ (d) $1/\sqrt{3}$

v. z lies in

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

Multiple Choice Questions

46. $(\sqrt{-2})(\sqrt{3})$ is equal to

(a) $\sqrt{6}$ (b) $-\sqrt{6}$

(c) $i\sqrt{6}$ (d) None of these

47. If $\frac{(a^2+1)^2}{2a-i} = x+iy$, x^2+y^2 is equal to

(a) $\frac{(a^2+1)^4}{4a^2+1}$ (b) $\frac{(a+1)^2}{4a^2+1}$

(c) $\frac{(a^2-1)^2}{(4a^2-1)^2}$ (d) None of these

48. If z = $\frac{1}{1-\cos\theta - i\sin\theta}$, then Re(z) =

(a) 0 (b) ½

(c) cot θ/2 (d) ½ cot θ /2

49. If f(z) = $\frac{7-z}{1-z^2}$, where z = 1 + 2i, then |f(z)| is

(a) $\frac{|z|}{2}$ (b) |z|

(c) 2|z| (d) None of these

50. The value of $(1+i)^4 + (1-i)^4$ is

(a) 8 (b) 4

(c) – 8 (d) – 4

51. The equation $|z+1-i| = |z-1+i|$ represent a

(a) Straight line (b) Circle

(c) Parabola (d) Hyperbola

52. The value of $\frac{i^{4n+1} - i^{4n-1}}{2}$ is

(a) 2i (b) –2i

(c) i (d) –i

53. If three complex number Z_1, Z_2 and Z_3 are in A.P, then points representing them lies on

(a) Circle (b) Parabola

(c) Hyperbola (d) Straight line

54. The sum of series $i + i^2 + i^3 + ...$ up to 1000 terms is

(a) 0 (b) *i*

(c) –*i* (d) None of these

55. If $z_1 = \sqrt{3} + i\sqrt{3}, z_2 = \sqrt{3} + i$, then the point $\frac{z_1}{z_2}$ lies in

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant

ANSWERS

1. $-1 + i$
2. $-7 - 6i$
3. $\frac{1}{49} - \frac{4\sqrt{3}i}{49}$
4. $\frac{-2}{25} + \frac{11i}{25}$
5. $\frac{-2}{7} - \frac{i\sqrt{3}}{7}$
6. 13
7. 7
8. –2
9. $-i$
10. First
11. –15
12. (i) 0

    (ii) 19

    (iii) –8

(iv) $\frac{-7}{\sqrt{2}}i$

13. $x = \frac{5}{13},\quad y = \frac{14}{13}$

14. i

15. 0 (zero)

16. 6 and zero

17. a = –2

19. $\theta = (2n+1)\frac{\pi}{2}$

21. x = 3, y = –1

23. zero

24. a = 0, b = –2

25. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

26. n = 4

28. (i) $3\sqrt{2}$ and $-2i$

(ii) –2i, –2i

29. ½, –2*i*

31. radius= $\frac{2}{3}$

32. $z = 0,\ i,\ \frac{\sqrt{3}}{2} - \frac{1}{2}i,\ -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

34. 1

35. K=1

36. Infinitely many solutions of the form $z = 0 \pm iy;\ y \in R$

37. $|z_1| = \sqrt{x^2 + y^2}$

38. 5

40. When $x = 1$, $y = -4$ or $x = -1$, $y = -4$

41. 1 (one)

44. 1

45. i. (a) ii. (b) iii. (d) iv. (c) v. (b)

46. (b)	47. (a)	48. (b)	49. (a)
50. (c)	51. (a)	52. (c)	53. (d)
54. (a)	55. (d)		

CHAPTER - 5

LINEAR INEQUALITIES

KEY POINTS

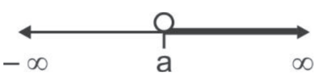
- **Inequalities:** A statement involving '<', '>','≥'or '≤'is called inequality.
  - Inequalities which do not involve variables are called numerical inequalities.
  - Inequalities which involve variables are called literal inequalities.
    Eg., $3x - 4 \leq 15$ and $4x - 3y \geq 5$
  - Inequalities involving the symbols '>' or '<' are called strict inequalities.
  - Inequalities involving the symbols '≥' or '≤' are called slack inequalities.
- **Linear inequalities in one variable:** The inequalities of form $ax + b > 0$, $ax + b < 0$, $ax + b \geq 0$ or $ax + b \leq 0$; $a \neq 0$ are called linear inequalities in one variable.

  The set of real numbers which satisfy a given linear inequality is called the solution set of the inequality.
- **Algebraic solutions of linear inequalities in one variables:**
  - **Rule-1**
    Equal numbers may be added (or subtracted from) to both sides without affecting sign of inequalities.

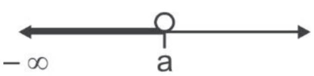
- **Rule-2**
  - (i) If both sides of inequality are multiplied (or divided) by same positive number, then sign of inequality remains unchanged.
  - (ii) If both sides are multiplied (or divided) by any negative number, then sign of inequality is reversed.

► **Graphical representation of solutions on number line:**

(i) $x > a \Leftrightarrow a < x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$

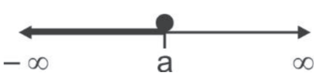


(ii) $x < a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$



(iii) $x \geq a \Leftrightarrow a \leq x < \infty \Leftrightarrow x \in [a, \infty) \Leftrightarrow$

(iv) $x \leq a \Leftrightarrow -\infty < x \leq a \Leftrightarrow x \in (-\infty, a] \Leftrightarrow$



(v) $a < x < b \Leftrightarrow x \in (a, b) \Leftrightarrow$

(vi) $a \leq x \leq b \Leftrightarrow x \in [a, b] \Leftrightarrow$

► **Linear inequalities in two variables:** The inequalities of form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$ or $ax + by + c \leq 0$ are linear inequalities in two variables. ($a, b \neq 0$)

Eg., $4x - 3y < 15$ and $-4x + 15y + 3 \geq 4$

► **Graphical solution of linear inequalities in two variables**

- A line divides the Cartesian plane into two parts. Each part is known as a half plane.

- The region containing all the solutions of the inequality is called solution region.
- In order to identify the half plane represented by an inequality (solution region), it is just sufficient to take any point (a, b) not on the line and check whether it satisfy the inequality or not.

  (i) If it satisfies, then the regions containing that point (a, b) is solution region.

  (ii) If it does not satisfy, then the other region is solution region.
- If inequality contains '$\geq$' or '$\leq$', then points on line $ax + by = c$ are also included in solution region. In this case we draw dark line while sketching graph of $ax + by = c$.
- If inequality contains '>' or '<', then points on line $ax + by = c$ are not included in solution region. In this case we draw dotted line while sketching graph of $ax + by = c$.

Note: While solving system of linear inequalities in two variables, the common of solution regions of each inequality is solution region of system.

VERY SHORT ANSWER TYPE QUESTIONS

1. Solve $5x < 24$ when $x \in N$
2. Solve $3 - 2x < 9$ when $x \in R$. Express the solution in the form of interval.
3. Show the graph of the solution of $2x - 3 > x - 5$ on number line.
4. Solve $0 < \frac{-x}{3} < 1, \quad x \in R$
5. Solve $-3 \leq -3x + 2 < 4, \quad x \in R$.

6. Draw the graph of the solution set of $x + y \geq 4$.

7. Draw the graph of the solution set of $x < y$.

8. Solve the inequality for real *x*: $\frac{x^2}{x-2} > 0$.

SHORT ANSWER TYPE QUESTIONS

9. Solve $\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0, \quad x \in R$.

10. Solve $\frac{x+3}{x-1} > 0, \quad x \in R$.

Solve the inequalities for real x and represent solution on number line

11. $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}, \quad x \in R$.

12. $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2, \quad x \in R$.

13. $-5 \leq \frac{2-3x}{4} \leq 9, \quad x \in R$.

14. $\frac{x+3}{x-2} > 0, \quad x \in R$

15. $\frac{x-3}{x-5} > 2$

16. $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$

17. $\frac{2x+3}{x-3} \le 4$

18. Find the pair of consecutive even positive integers which are greater than 5 and are such that their sum is less than 20.

19. A company manufactures cassettes and its cost and revenue functions are C(x) = 26000 + 30x and R(x) = 43x respectively, where x is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realise some profit.

20. While drilling a hole in the earth, it was found that the temperature (T°C) at x km below the surface of the earth was given by T = 30 + 25(x – 3), when$3 \le x \le 15$.

    Between which depths will the temperature be between 200°C and 300°C?

21. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3rd reading that will result in acidity level being normal.

Solve the following systems of inequalities for all x $\in$ R

22. $2(2x+3)-10<6(x-2), \quad \frac{2x-3}{4}+6 \ge 4+\frac{4x}{3}$

23. $|2x-3| \le 11, \quad |x-2| \ge 3$

24. $\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4}, \quad \frac{7x-1}{3}-\frac{7x+2}{6}>x$

25. Solve $\frac{|x|-1}{|x|-2} \ge 0 \quad x \in R, \quad x \ne \pm 2$

26. In the first four papers each of 100 marks, Rishi got 95, 72, 73, 83 marks. If he wants an average of greater than or equal to 75 marks be should score in fifth paper.

27. A milkman has 80% milk in this stock of 800 litres of adultered milk. How much 100% pure milk is to be added to it so that purity is between 90% and 95%?

28. $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}, \ \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

29. $\frac{x}{2x+1} \geq \frac{1}{4}, \ \frac{6x}{4x-1} < \frac{1}{2}$

30. $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 45$.

LONG ANSWER TYPE QUESTIONS

Solve following system of inequalities graphically.

31. $2x + y \leq 24$, $x + y < 11$, $2x + 5y \leq 40$, $x \geq 0$, $y \geq 0$

32. $3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$

33. $x - 2y \leq 3$

    $3x + 4y > 12$

    $x \geq 0$, $y \geq 1$

CASE STUDY TYPE QUESTIONS

34. A company produced cassettes, one cassette Cost Company Rs. 30 and also an additional fixed cost 26000 per week. The company sold each Cassette at Rs. 43. If x is number of

cassettes produced and sold by the company in a week. From the following information find

i. The cost function of the company

(a) 26000 + 30x (b) 26000+43x

(c) 30 +26000x (d) 43 + 26000x

ii. The revenue function of the company

(a) 30x (b) 26000x

(c) 43x (d) 13x

iii. The profit function of the company

(a) –26000 + 73x (b) –26000+ 13x

(c) 26000+43x (d) 26000+ 30x

iii. How many cassettes must be produced by the company in a week to realize some profit?

(a) more than 2000 (b) less than 2000

(c) more than 5000 (d) less than 5000

iv. If company incurred an additional cost of Rs. 3 on each cassette per week. How manycassettes must be produced by the company in a week so that there is no profit no loss?

(a) 2000 (b) 5000

(c) 2600 (d) 1000

35. A and B try to find the solution of the inequality $|x - 1| + |x - 2| \geq 4$. Help them to find the solution of the inequality

i. When $x < 1$

(a) $(-\infty, -1/2)$ (b) $(-\infty, -1)$

(c) $(-\infty, -1/2]$ (d) $(-\infty, 1/2)$

ii. When $1 \le x < 2$

(a) $(-\infty, \infty)$ (b) $(-\infty, -1)$

(c) Infinite solution (d) no solution

iii. When $2 \le x < \infty$

(a) $(-\infty, \infty)$ (b) $(7/2, \infty)$

(c) $[7/2, \infty)$ (d) no solution

iv. When $x \in R$

(a) $(-\infty, -1/2] \cup [7/2, \infty)$ (b) $(-\infty, -1/2) \cup [7/2, \infty)$

(c) $(-\infty, -1/2] \cup (7/2, \infty)$ (d) $(-\infty, -7/2] \cup [1/2, \infty)$

v. When $x > 4$

(a) $(-\infty, 4)$ (b) $(-\infty, 4]$

(c) $(4, \infty)$ (d) $[4, \infty)$

Multiple Choice Questions

36. If $-4x > 20$ and $x \in z^+$ then x belongs to -

(a) $\{-6, -7, -8, \ldots\ldots.\}$ (b) ϕ

(c) $\{-4, -3, -2, -1\}$ (d) $\{1, 2, 3, 4, \ldots\ldots.\}$.

37. If $\dfrac{x-3}{x-2} > 0$ then x belongs to -

(a) $(-\infty, 3) \cup (5, \infty)$ (b) $(-\infty, -3) \cup (-5, \infty)$

(c) $(-\infty, 3] \cup [5, \infty)$ (d) $(3, 5)$

38. Solution set for inequality $|x - 1| \leq 5$ is -

(a) $[-6, 4]$ (b) $[-4, 0]$

(c) $[-4, 6]$ (d) $[0, 6]$.

39. Solution set for inequality $\frac{1}{x-2} < 0$ is -

(a) $(2, \infty)$ (b) ϕ

(c) $(0, 2)$ (d) $(-\infty, 2)$.

40. Solution set for inequality $5x - 3 < 3x + 1$, $x \in N$ is -

(a) $(-\infty, 2)$ (b) $\{0, 1, 2\}$

(c) $\{1\}$ (d) ϕ.

41. Which of the following point lies in solution region of inequality $3x - y \leq 5$?

(a) $(5, 1)$ (b) $(1, 5)$

(c) $(2, 0)$ (d) $(2, -1)$.

42. If $x > 0$ and $y < 0$ then (x, y) lies in -

(a) I quadrant (b) II quadrant

(c) III quadrant (d) IV quadrant.

43. If $x^2 > 9$ then x belongs to -

(a) $(-3, 3)$ (b) $(0, 3)$

(c) $(3, \infty)$ (d) $(-\infty, -3) \cup (3, \infty)$.

44. Solution set for inequality $-8x \leq 5x - 3 < 7$ is -

(a) $(-1, 2)$ (b) $(2, 3)$

(c) $[-1, 2)$ (d) $[2, 3]$.

45. The graph of the inequalities

$x \leq 0$, $y \geq 0$, $2x + y + 6 \leq 0$ is

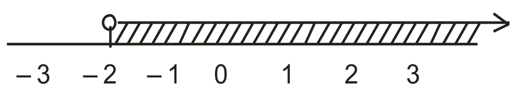
(a) a triangle (b) a square

(c) { } (d) none of these

ANSWERS

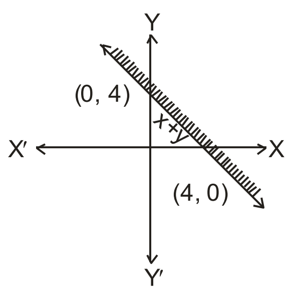
1. {1,2,3,4}

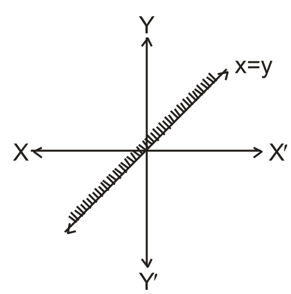
2. $(-3, \infty)$

3. 

4. $-3 < x < 0$

5. $\left(\frac{-2}{3}, \frac{5}{3}\right]$

6. 



8. $[2, \infty)$

9. $(-\infty, 1] \cup [2, 3) \cup (4, \infty)$

10. $(-\infty, -3) \cup (1, \infty)$

11. $\left(-\infty, \frac{63}{10}\right]$

12. $\left(-\infty, \frac{-13}{2}\right)$

13. $\left[\frac{-34}{3}, \frac{22}{3}\right]$

14. $(-\infty, -3) \cup (2, \infty)$

15. (5, 7)

16. $(-\infty, 2]$

17. $(-\infty, -3) \cup (2, \infty)$

18. (6, 8) and (8, 10)

19. More than 2000 cassettes

20. Between 9.8 m and 13.8 m

21. Between 6.27 and 8.07.

22. Solution set $=\phi$

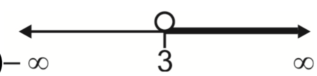
23. $[-4, -1] \cup [5, 7]$

24. (4, 9)

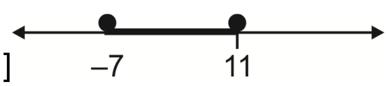
25. $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

26. He must score greater than or equal to 52 and less than 77.

27. Between 100 litre and 150 litre

28. $(3, \infty)$ 

29. Number solution

30. $[-7, 11]$ 

34. i. (a) ii. (c) iii. (b) iv. (a) v. (c)

35. i. (c) ii. (d) iii. (c) iv. (a) v. (c)

36. (b) 37. (a)

38. (c) 39. (d)

40. (c) 41. (b)

42. (d) 43. (d)

44. (c) 45. (c)

CHAPTER - 7

PERMUTATIONS AND COMBINATIONS

KEY POINTS

▶ **Fundamental principal of counting**

- **Multiplication Principle:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of simultaneous occurrence of the two events in order is m × n.

- **Fundamental Principle of Addition:** If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in (m + n) ways.

▶ **Factorial:** Factorial of a natural number n, denoted by n! or n is the continued product of first n natural numbers.

$n! = n \times (n - 1) \times (n - 2) \times \ldots\ldots\ldots \times 3 \times 2 \times 1$

$= n \times ((n - 1)!)$

▶ **Permutation:**A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.

- The number of permutation of n different objects taken r at a time where$0 \leq r \leq n$ and the objects do not repeat is denoted by ${}^nP_r$or P(n, r) where,

$${}^nP_r = \frac{n!}{(n-r)!}$$

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, …….. p_k are of k^{th} kind and the rest if any, are of different kinds, is $\frac{n!}{(p_1!)(p_2!)\,.......\,(p_k!)}$

▶ **Combination:** Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n distinct objects taken r at a time where,

$0 \le r \le n$, is denoted by ${}^nC_r$ or C(n, r) where ${}^nC_r = \frac{n!}{r!(n-r)!}$

▶ **Some important result:**

(i) $0! = 1$

(ii) ${}^nC_0 = {}^nC_n = 1$

(iii) ${}^nC_r = {}^nC_{n-r}$ where $0 \le r \le n$, and r are positive integers

(iv) ${}^nP_r = \underline{|n}\,{}^nC_r$ where $0 \le r \le n$, r and n are positive integers.

(v) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

(vi) If ${}^nC_a = {}^nC_b$ if either a = b or a + b = n

VERY SHORT ANSWER TYPE QUESTIONS

1. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If ${}^nP_4 : {}^nP_2 = 12$, find n.

7. How many different words (with or without meaning) can be made using all the vowels at a time?

8. In how many ways 4 boys can be chosen from 7 boys to make a committee?

9. How many different words can be formed by using all the letters of word "SCHOOL"?

10. In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

11. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

12. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

13. If ${}^{20}C_r = {}^{20}C_{r-10}$ then find the value of ${}^{18}C_r$

SHORT ANSWER TYPE QUESTIONS

14. Find n, ${}^{n-1}P_3 : {}^{n}P_4 = 1 : 9$.

15. If ${}^{22}P_{r+1} : {}^{20}P_{r+2} = 11 : 52$, find r.

16. If ${}^{n}P_r = 336$, ${}^{n}C_r = 56$, find n and r. Hence find ${}^{n-1}C_{r-1}$.

17. A convex polygon has 65 diagonals. Find number of sides of polygon.

18. In how many ways can a cricket team of 11 players be selected out of 16 players, if two particular players are always to be selected?

19. From a class of 40 students, in how many ways can five students be chosen for an excursion party.

20. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together?

21. If ${}^{n}C_{12} = {}^{n}C_{13}$ then find the value of the ${}^{25}C_n$.

22. In how many ways can the letters of the word "PENCIL" be arranged so that I is always next to L.

23. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice.

24. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

25. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

26. A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?

27. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

28. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

29. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

30. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

31. A polygon has 35 diagonal. Find the number of its sides.

32. Determine the number of 5 cards combinations out of a pack of 52 cards if at least 3 out of 5 cards are ace cards?

33. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

34. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.

35. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has

    (i) no girl

(ii) at least 3 girls

(iii) at least one girl and one boy?

36. In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter vote for at least one candidate, then find the number of ways in which he can vote.

37. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

LONG ANSWER TYPE QUESTIONS

38. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

39. There are 15 points in a plane out of which only 6 are in a straight line, then

    (i) How many different straight lines can be made?

    (ii) How many triangles can be made?

40. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that

    (i) No two girls sit together?

    (ii) All the girls never sit together?

41. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

42. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

(i) 3 are red and 1 is black.

(ii) All 4 cards are from different suits.

(iii) Atleast 3 are face cards.

(iv) All 4 cards are of the same colour.

43. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

44. How many different four letter words can be formed (with or without meaning) using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R.

45. If all letters of word 'MOTHER' are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word 'MOTHER'?

46. From 6 different novels and 3 different dictionaries, 4 novels and a dictionary is to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then find the number of such arrangements.

47. The set S= {1,2,3,.......12} is to be partitioned into three sets A, B, and C of equal sizes. $A \cup B \cup C = S$, $A \cap B = B \cap C = C \cap A = \phi$. Find the number of ways to partition S.

48. Find the value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$.

CASE STUDY TYPE QUESTIONS

49. Anita is doing an experiment in which she has to arrange the alphabets of the word "HARYANA" in all possible orders and notes the observations. Help her to find the answers ofthe following:-

 i. Number of words starting with A

 (a) 360 (b) 720

 (c) 1440 (d) 2880

 ii. Number of words having H at end

 (a) 72 (b) 120

 (c) 240 (d) 480

 iii. Number of words having Hand N together

 (a) 120 (b) 60

 (c) 280 (d) 240

 iv. Number of words having begin with H and end with N

 (a) 0 (b) 24

 (c) 60 d) 48

 v. Number of words having vowels together

 (a) 240 (b) 120

 (c) 240 (d) 720

50. A Company wants to appoint 5 persons, 3 for post A and 2 for post B for its upcoming officein Delhi. They have invited the applications for the same. 14 candidates have applied for the postA and 13 have applied for the post B

i. Find the total number of ways in which the company can make a selection for all the posts.

(a) 5! (b) C(14,3).C(13,2)

(c) P(13,2)P(14,3) (d) none of these

ii. Find the number of ways of selecting one woman for each post, if 3 women have applied for post A and 7 women have applied for post B

(a) 6 (b) 21

(c) 930 (d) 182

iii. On the day of interview, the candidates were seated in a hall having two chambers. The chairs in both the chambers are placed in line. If the candidates for the two posts are to be seated in two different chambers. Find the total number of ways in which all the candidate can be seated.

(a) 3!2! (b) 11!11!

(c) 14!13! (d) 14! × 13! × 2

iv. During appointment procedure they came to know about a candidate whose resume is excellent and should be selected for the post B. In how many ways can the total selections now be made?

(a) 12 × C(14,3) (b) 4

(c) 168 (d) 13 × C(13,3)

v. While checking the applications the management observed that one candidate each who have applied for post A and B are not fit for the job so they cannot be appointed in how many ways can now the post is filled?

(a) 2184 (b) 24024

(c) 18876 (d) 1716

Multiple Answer Type Questions

51. What is the number of ways of arrangement of letters of word 'BANANA' so that no two N's are together -

(a) 40 (b) 60

(c) 80 (d) 100.

52. What is the value of n, if P(15, n – 1) : P(16, n – 2) = 3 : 4 ?

(a) 10 (b) 12

(c) 14 (d) 15.

53. The number of words which can be formed from the letters of the word MAXIMUM, if two consonants can't occur together is -

(a) 4! (b) 3! × 4!

(c) 7! (d) None of these.

54. If 7 points out of 12 are in the same straight line, then what is the number of triangles formed?

(a) 84 (b) 175

(c) 185 (d) 201

55. In how many ways can be bowler take four wickets in a single 6 balls over?

(a) 6 (b) 15

(c) 20 (d) 30.

56. What is the number of signals that can be sent by 6 flags of different colours taking one or more at a time?

(a) 45 (b) 63

(c) 720 (d) 1956.

57. There are 6 letters and 3 post boxes. The number of wages in which these letters can be posted is –

(a) 6^3 (b) 3^6

(c) ${}^6P_3$ (d) ${}^6C_3$.

58. If ${}^mC_1 = {}^nC_2$, then –

(a) $2m = n$ (b) $2m = n(n + 1)$

(c) $2m = n(n - 1)$ (d) $2n = m(m - 1)$.

59. ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_x$, then x = ?

(a) r (b) r – 1

(c) n (d) r + 1.

60. ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$, then value of r is –

(a) 12 (b) 8

(c) 6 (d) 10.

ANSWERS

1. $\frac{6!}{2} = 360$ 2. $\frac{9!}{2!}$

3. 100 4. 45

5. 120 6. n = 6

7. 120 8. 35

9. 360 10. 63

11. $3^6=729$ 12. 66

13. 816

14. n = 9

15. r = 7

16. n = 8, r = 3 and 21

17. 13

18. 2002

19. 40_{C_5} 20. $\frac{3!}{2!} \times 4!$

21. 1 22. 120

23. $90 \times {}^{10}P_8$ 24. 56

25. 350 26. 266

27. 36 28. PAANVR

29. ${}^{13}C_{10} + {}^{13}C_8$ 30. 5040

31. 10 32. 4560

33. 576

34. 2454

35. (i) 21;

(ii) 91;

(iii) 441

36. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

37. 48,144

38. 420

39. (i)91

(ii)435

40. (i)$7! \times {}^{8}P_5$

(ii)$12! - 8! \times 5!$

41. 24480

42. ${}^{52}C_4$

(i) ${}^{26}C_1 \times {}^{26}C_3$

(ii) $(13)^4$

(iii) 9295 (Hint : Face cards : 4J + 4K + 4Q)

(iv) $2 \times {}^{26}C_4$

43. 265 (*Hint* : make 3 cases i.e.

(i) All 3 letters are different

(ii) 2 are identical 1 different

(iii) All are identical, then form the words.)

44. 59

45. 309

46. $4!\,{}^{6}C_4\,{}^{3}C_1$

47. ${}^{12}C_4\,{}^{8}C_4\,{}^{4}C_4$

48. ${}^{56}C_4$

49. i. (a) ii. (b) iii. (c) iv. (a) v. (b)

50. i. (b) ii. (c) iii. (d) iv. (a) v. (c)

51. (a)

52. (c)

53. (a)

54. (c)

55. (b)

56. (b)

57. (b)

58. (c)

58. (d)

60. (a)

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

- **Binomial Theorem for Positive Integers :**
  - $(x + y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots\ldots\ldots$

    $\ldots\ldots\ldots\ldots + {}^nC_r x^{n-r} y^r + \ldots\ldots\ldots\ldots {}^nC_2 x^0 y^n,$

    Where n is any positive integer.
  - It is written as $(x+y)^n = \sum_{r=0}^{n} {}^nc_r x^{n-r} y^r$
  - Total number of terms in expansion $(x + y)^n$ is $(n + 1)$
  - General Term = $T_{r+1} = {}^nC_r x^{n-r} y^r$, where $0 \le r \le n$.

- **Middle Term :**
  - If n is even, then there is only one middle term

    M.T. = $\left(\frac{n}{2}+1\right)^{th}$ term
  - If n is odd, then there are two middle terms

    First M.T. = $\left(\frac{n+1}{2}\right)^{th}$ term

    Second M.T. = $\left(\frac{n+1}{2}+1\right)^{th}$ term

▶ **Some important observations :**

- In expansion $(x + y)^n$

  ${}^nc_r, {}^nc_{r-1}, \ldots {}^nc_0$ are called **binomial coefficients**

- Sum of indices of x and y is n in each of the expansion.

- T_{r+1} = [(r + 1)th term from beginning] = ${}^nC_r x^{n-r} y^r$

- T'_{r+1} = [(r + 1)th term from end] = ${}^nC_{n-r} x^r y^{n-r}$

- $(x-y)^n = \Sigma(-1)^r \; {}^nC_r \; x^{n-r} y^r$

- $(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$

- $(1-x)^n = \sum_{r=0}^{n} (-1)^r \; {}^nC_r x^r$

VERY SHORT ANSWER TYPE QUESTIONS

1. Write number of terms in the expansion of $\left\{\left(2x+y^3\right)^4\right\}^7$.

2. Expand $\left(\sqrt{\frac{x}{a}}-\sqrt{\frac{a}{x}}\right)^6$ using binomial theorem.

3. Write value of

   ${}^{2n-1}c_5 + {}^{2n-1}c_6 + {}^{2n}c_7$ use $\left[{}^nc_r + {}^nc_{r-1} = {}^{n+1}c_r\right]$

4. Which term is greater $(1.2)^{4000}$ or 800?

5. Find the coefficient of x^{-17},in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$.

6. Find the sum of the coefficients in $(x + y)^8$

[**Hint :** Put x = 1, y = 1]

7. If ${}^nC_{n-3} = 720$, find n.

8. Find number of terms in expansion of $(-2x + 3y)^{17}$.

9. Find term independent of x in expansion of $\left(x - \frac{1}{3x^2}\right)^9$

10. Find the middle term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$

11. If the coefficient of x in $\left(x^2 + \frac{\lambda}{x}\right)^5$ is 270, then find λ.

12. Find the coefficient of x^5 in $(x + 3)^8$

13. Find 4th term from the end in expansion of $\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$

14. Find number of terms in $(x + y)^5 + (x - y)^5$

15. Find coefficient f x^5 in $(1 + x)^{10}$

SHORT ANSWER TYPE QUESTIONS

16. How many term are free from radical signs in the expansion of $\left(x^{\frac{1}{5}} + y^{\frac{1}{10}}\right)^{55}$.

17. Find the constant term in expansion of $\left(x - \frac{1}{x}\right)^{10}$.

18. Find 4th term from end in the expansion of find the value of $\left(\frac{x^3}{2}-\frac{2}{x^2}\right)^9$.

19. Find middle term in the expansion of $(x - 2y)^8$.

20. Which term is independent of x in the expansion of $\left(3x^3-\frac{1}{2x^3}\right)^{10}$.

21. Find the 11th term from end in expansion of $\left(2x-\frac{1}{x^2}\right)^{25}$.

22. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

23. In $\left(3x^2-\frac{1}{x}\right)^{18}$ which term contains x^{12}.

24. In $\left(\frac{\sqrt{x}}{\sqrt{3}}+\frac{\sqrt{3}}{\sqrt{2}x^2}\right)^{10}$ find the term independent of x.

25. Evaluate $\left(\sqrt{2}+1\right)^5-\left(\sqrt{2}-1\right)^5$ using binomial theorem.

26. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

27. Find the coefficients of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

28. Show that$3^{2n+2} - 8n - 9$ is divisible by 8.

29. If the term free from x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405. Find the value of k.

30. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}}+7^{\frac{1}{8}}\right)^{1024}$.

31. If a, b, c and d in any binomial expansion be the 6^{th}, 7^{th}, $8^{th}$and 9^{th}terms respectively, then prove that $\frac{b^2-ac}{c^2-bd}=\frac{4a}{3c}$.

32. If in the expansion of $(1+x)^n$, the coefficients of three consecutive of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.

33. Show that $2^{4n+4} - 15n - 16$ where $n \in N$ is divisible by 225.

34. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

35. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$.

36. Find the value of r, if the coefficient of $(2r + 4)^{th}$ term and $(r - 2)^{th}$ term in the expansion of $(1 + x)^{18}$ are equal.

37. Prove that there is no term involving x^6 in the expansion of $\left(2x^2-\frac{3}{x}\right)^{11}$.

38. The coefficient of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 6 : 30. Find n.

LONG ANSWER TYPE QUESTIONS

39. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.

40. If the 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n.

41. If the coefficients of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ are equal, then show that ab = 1.

42. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is 1:6 , find n.

43. If p is a real number and if middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, find p.

44. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms inthe expansion of $(1 + x)^n$

Prove that $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$.

45. Using binomial theorem, find the remainder when $5^{103}$is divided by 13.

46. Find the remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9.

47. Find the coefficient of x^n in expansions of $(1 + x)(1 - x)^n$.

48. Find the value of $\left(\sqrt{2}+1\right)^6 - \left(\sqrt{2}-1\right)^6$ and show that $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

49. Find the term independent of x in the expansion of : $(1 + x + 2x^3)$ $\left(\frac{3}{2}x^2 - \frac{1}{3}x\right)^9$.

50. If the coefficients of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms in the expansion of $(1 + x)^4$ are in A.P find the value of r.

51. If the expansion of $(1 - x)^{2n-1}$, the coefficients of x^r is denoted by a_r, then prove $a_{(r-1)} + a_{(2n-r)} = 0$.

52. If the coefficient of 5^{th}, 6^{th} and 7^{th} terms in the expansion of $(1 + x)^n$ are in A.P., then find the value of n.

53. The coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(1 + x)^{2n}$ are in A.P. Prove that $2n^2 - 9n + 7 = 0$.

54. Show that the middle term in the expansion of $\left[x - \frac{1}{x}\right]^{2n}$ is $\frac{1 \cdot 3 \cdot 5.....(2n-1)}{n!}(-2n)^n$.

55. If *n* is a positive integer, find the coefficient of x^{-1} in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$

Multiple Choice Questions

56. The middle term of $\left[2x - \frac{1}{3x}\right]^{10}$ is -

(a) ${}^{10}C_4 \frac{2^4}{3^4}$ (b) $-{}^{10}C_5 \frac{2^5}{3^5}$

(c) $-{}^{10}C_4 \frac{2^4}{3^5}$ (d) ${}^{10}C_5 \frac{2^5}{3^5}$.

57. For all $n \in N$, $2^{4m} - 15n - 1$ is divisible by -

(a) 125 (b) 225

(c) 450 (d) 625.

58. What is the coefficient of x^n in $(x^2 + 2x)^{n-1}$?

(a) $(n - 1)\,2^{(n-1)}$ (b) $(n - 1) \times 2^{(n-1)}$

(c) $(n - 1)\,2^n$ (d) $n \,.\, 2^{(n-1)}$.

59. The coefficient of x^{-3} in the expansion of $\left[x - \frac{m}{x}\right]^{11}$ is –

(a) $-924\,m^7$ (b) $-792\,m^5$

(c) $-792\,m^6$ (d) $-330\,m^7$.

60. In the expansion of $\left[x^2 - \frac{1}{3x}\right]^9$, the term without x is equal to -

(a) $\frac{28}{81}$ (b) $\frac{-28}{243}$

(c) $\frac{28}{243}$ (d) None of these.

61. If in the expansion of $(1 + x)^{20}$, the coefficients of r^{th} and $(r + 4)^{th}$ term are equal, then x is equal to -

(a) 7 (b) 8

(c) 9 (d) 10.

62. If in the expansion of $(1 + x)^5$, the coefficients of $(r - 1)^{th}$ and $(2r + 3)^{th}$ terms are equal, then the value of x -

(a) 5 (b) 6

(c) 4 (d) 3.

63. The total number of terms in expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is -

(a) 202 (b) 51

(c) 50 (d) None of these.

64. The middle term in the expansion of $\left[\frac{2x}{3} - \frac{3}{2x^2}\right]^{2n}$ is -

(a) ${}^{2n}C_n$

(b) $(-1)^{n\,2n}C_n\, x^{-n}$

(c) ${}^{2n}C_n\, x^{-n}$

(d) None of these.

65. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are the some, then the value of a is -

(a) $\frac{-7}{9}$

(b) $\frac{-9}{7}$

(c) $\frac{7}{9}$

(d) $\frac{9}{7}$.

ANSWERS

1. 29

2. $\frac{x^3}{a^3}-\frac{6x^2}{a^2}+15\frac{x}{a}-20+15\frac{a}{x}-\frac{6a^2}{x^2}+\frac{a^3}{x^3}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. –1365

6. 256

7. n = 10

8. 18

9. ${}^{9}C_3 \times \left(\frac{-1}{3}\right)^3$

10. ${}^{10}C_5$

11. 3

12. 152

13. $\frac{672}{x^3}$

14. 3

15. ${}^{10}C_5$

16. 6 terms (0, 10, 20, 30, 40, 40, 50)

17. $-252 = -{}^{10}C_5$

18. $\frac{672}{x^3}$

19. $1120\ x^4\ y^4$

20. $\frac{-15309}{8}$

21. $^{25}C_{15} \times \frac{2^{10}}{x^{20}}$

22. a = 3, b = 2, n = 3

23. 9^{th} term

24. $T_3 = \frac{5}{6}$

25. 82

26. 28

27. 10

29. $k = \pm 3$

30. 129 integral terms

32. n = 8, 4^{th},$5^{th}$and 6^{th}

36. r = 6

37. $\left(2x^2 - \frac{3}{x}\right)^{11}$

38. n = 41

40. a = 2, x = 1, n = 7

42. 9

45. 8

46. 2

47. $(-1)^n\ [1 - n]$

48. Zero

49. $\frac{17}{54}$

50. 5

52. n = 7 or 14

55. $^{2n}C_{n-1}$

56. (b)

57. (b)

58. (a)

59. (d)

60. (c)

61. (c)

62. (a)

63. (b)

64. (b)

65. (d)

CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

- In general, listing of any collection of objects in certain order is **sequence**.
- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- Let a_1, a_2, a_3, ……….. be a sequence, then the expression $a_1 + a_2 + a_3 +$ ……….. is called series associated with given sequence.
- A sequence containing finite number of terms is called finite sequence.
- A sequence is infinite, if it is not finite sequence.
- A sequence is said to be a progression if all the terms of the sequence can be expressed by same formula
- **Arithmetic Progression:** A sequence is called an arithmetic progression if the difference between of a term and its previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=d) for all $n \in N$.
- General A.P. isa, a + d and a + 2d, ..., where a = first term and d = common difference.
- $a_n = a + (n - 1)d = n^{th}$ term of A.P. $= l$
- S_n = Sum of first n terms of A.P. $= \frac{n}{2}[a + l]$, where l = last term N.

$$= \frac{n}{2}[2a + (n-1)d]$$

- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P.

  ak, bk, ck also in A.P., $k \neq 0$

  $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where $k \neq 0$.

- If a, A, b are in A.P., then A is called **arithmetic** mean of a and b.

- Arithmetic mean between a and b is = $\frac{a+b}{2}$.

- If $A_1, A_2, A_3, ...A_n$ are n numbers inserted between a and b, such that the resulting sequence is A.P.

  then, $A_n = a + n \cdot \frac{b-a}{n+1}$

- $S_k - S_{k-1} = a_k$

- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term.

- If a, b, c are in A.P. then 2b = a + c.

- Three terms of A.P. can be chosen as a – d, a, a + d

- Four terms of A.P. can be chosen as a – 3d, a – d, a + d, a + 3d.

- **G.P. (Geometrical Progression)**

  (i) a, ar, ar^2, ...(General G.P.)

  Where a = First term

  And r = common ratio

(ii) $a_n = ar^{n-1}$

(iii) $S_n = \dfrac{a(r^n - 1)}{r - 1}$ or $S_n = \dfrac{a(1-n^n)}{1-r}$, $r \neq 1$

- If a, b, c are in G.P., then $b^2 = ac$.
- If a, G, b are in GP, then G is called **geometric** mean of a and b
- Geometric mean of two positive numbers a and b is $\sqrt{ab}$.
- If $G_1, G_2, G_3, \ldots\ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $G_k = a\left(\dfrac{b}{a}\right)^{\frac{k}{k+1}}$ $1 \leq k \leq n$

- Three terms of G.P. are chosen as $\dfrac{a}{r}$, a, ar.
- Four terms of G.P. are chosen as $\dfrac{a}{r^3}$, $\dfrac{a}{r}$, a, ar^3.
- If a, b, c are in G.P. then (i) $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are also in GP, (ii) ak, bk, ck are also in G.P., where k ≠ 0 (iii) $\dfrac{a}{k}$, $\dfrac{b}{k}$, $\dfrac{c}{k}$ are also in G.P. where k ≠ 0, a^n, b^n, c^n are also in GP.
- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.

- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1-r}$.

VERY SHORT ANSWER TYPE QUESTIONS

1. If n^{th} term of an A.P. is $6n - 7$ then write its 50^{th} term.

2. If $S_n = 3n^2 + 2n$, then write a_2

3. Which term of the sequence 3, 10, 17, ... is 136?

4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.

6. Which term of the G.P. $2, 1, \frac{1}{2}, \frac{1}{4}$ .... is $\frac{1}{1024}$?

7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.

8. In G.P. $2\sqrt{2}, 4, \ldots\ldots, 128\sqrt{2},$ find the 4^{th} term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term.

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}, \ldots\ldots\ldots$

11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ .... upto infinity.

12. Write the value of $0.\overline{3}$

    [Hint: $0.\overline{3} = 0.3 + 0.03 + 0.003 + \ldots = \frac{0.3}{1-0.1}$]

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

14. If 7^{th} and 13^{th} terms of an A.P. be 34 and 64 respectively, find 18^{th} term.

15. Find geometric mean of 4 and 9.

16. Find If the sum of first p terms of an A.P. is q and sum of first q terms is p, then the sum of first p + q terms.

17. Find sum to infinity of sequence $5, \frac{5}{3}, \frac{5}{9}, \ldots\ldots\ldots$

18. If a, b, c are in A.P. and x, y, z are in G.P., then find the value of $x^{b-c} \times y^{c-a} \times z^{c-a}$.

19. Find two geometric means between numbers 1 and 64.

20. Write third term of sequence whose general term is $a_n = \frac{2n-3}{4}$.

SHORT ANSWER TYPE QUESTIONS

21. Write the n^{th} term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + \ldots\ldots\ldots$

22. Find the number of terms in the A.P. 7, 10, 13, ..., 31.

23. In an A.P.,

    8, 11, 14, ... find $S_n - S_{n-1}$

24. Find the sum of given terms:-

    (a) 81 + 82 + 83 .... + 89 + 90

    (b) 251 + 252 + 253 + ... + 259 + 260

25. (a) If a, b, c are in A.P. then show that 2b = a+c.

    (b) If a, b, c are in G.P. then show that $b^2 = a \cdot c$.

26. If a, b, c are in G.P. then show that $a^2 + b^2$, $ab + bc$, $b^2 + c^2$ are also in G.P.

27. Find the least value of n for which

$1 + 3 + 3^2 + ... + 3^{n-1} > 1000$

28. Write the first negative term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$

29. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11^{th} term from the end.

30. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

31. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

32. Find the sum of the sequence,

$72 + 70 + 68 + \dots\dots\dots + 40$

33. If in an A.P $\frac{a_7}{a_{10}} = \frac{5}{7}$, find $\frac{a_4}{a_7}$.

34. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

35. Solve: $1 + 6 + 11 + 16 + ... + x = 148$

36. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their 10^{th} terms.

37. If the I^{st}, 2^{nd} and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

38. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{a}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term] abc.

39. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

40. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

41. Find the sum to infinity of the series:

$$1 + \frac{3}{2} + \frac{5}{2^2} + \frac{7}{2^3} + \text{......} \infty$$

42. If A = 1 + r^a + r^{2a} + ... up to infinity, then express r in terms of 'a' and 'A'.

43. Find the sum of first terms of the series 0.7 + 0.77 + 0.777 + …..

44. If $x = a + \frac{a}{r} + \frac{a}{r^2} + \text{...........} + \infty$; $y = b - \frac{b}{r} + \frac{b}{r^2} - \text{...........} \infty$ and

$z = c + \frac{c}{r^2} + \frac{c}{r^4} + \text{........} \infty$ Prove that $\frac{xy}{z} = \frac{ab}{c}$.

45. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

46. Prove that $0.003\overline{1} = \frac{7}{225}$.

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

47. If a, b, c are in G.P. that the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

48. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b+c, c+a, a+b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

49. If the numbers a^2, b^2 and c^2 are given to be in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in A.P.

50. Show that: $0.3\overline{56} = \frac{353}{990}$

51. The n^{th} term of a G.P. is 128 and the sum of its n term is 225. If its common ratio is 2, find the first term.

52. The fourth term of a G.P. is 4. Find product of its first seven terms.

53. If A_1, A_2, A_3, A_4 are four A.M's between $\frac{1}{2}$ and 3, then prove $A_1 + A_2 + A_3 + A_4 = 7$.

54. If S_n denotes the sum of first n terms of an A.P. If $S_{2n} = 5S_n$, then prove $\frac{S_{6n}}{S_{3n}} = \frac{17}{4}$.

LONG ANSWER TYPE QUESTIONS

55. Prove that the sum of n numbers between a and b such that the resultingseries becomes A.P. is $\frac{n(a+b)}{2}$.

56. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 15 cm, then find the sum of the areas of all the squares so formed.

57. If a, b, c are in G.P., then prove that $\frac{1}{a^2-b^2} - \frac{1}{b^2-c^2} = -\frac{1}{b^2}$.

    [**Hint :** Put b = ar, c = ar^2]

58. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

59. If a is A.M. of b and c and c, G_1, G_2, b are in G.P., then prove that $G_1^3 + G_2^3 = 2abc$

60. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

61. Find the sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ terms.

62. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

63. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 91.

CASE STUDY TYPE QUESTIONS

64. Abhishek buys Kisan Vikas Patra (KVP) from post office every year. Each year he exceeds the value of KVP by ₹1000 from last year's purchase. After 5 years he finds that the total value of KVP purchased by him is ₹40,000.00.

Based on the above information answer the following :-

i. The sequence of amount of KVP forms a/an

(a) Anithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the amount of KVP purchased by him initially.

(a) ₹7000 (b) ₹8000

(c) ₹6000 (d) ₹7500

iii. What will be the total amount of KVP purchased by him after 10 years?

(a) ₹1,20,000 (b) ₹1,05,000

(c) ₹1,40,000 (d) ₹1,35,000

iv. What is the amount of KVP purchased by him in the 8^{th} year?

(a) ₹14,000 (b) ₹15,000

(c) ₹13,000 (d) ₹12,000

v. If he buys KVP every year for 10 years, how much will he spend in the purchase of last 4 KVP?

(a) ₹65,000 (b) ₹54,000

(c) ₹75,000 (d) None of these

65. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail it to four different persons with the instruction that they move the chain similarly. Assuming that

the chain is notbroken and that it costs 50 paisa to mail one letter, anwer the following questions.

i. The sequence of letters mailed in each set forms a/an.

(a) Anithmetic Progression (b) Geometric Progression

(c) Harmonic Progression (d) None of these

ii. Find the number of letters mailed in the 4^{th} set.

(a) 64 (b) 16

(c) 256 (d) 1024

iii. Find the total number of letters mailed in the first 5 sets.

(a) 1364 (b) 1650 (c) 1236 (d) 1368

iv. Find the amount spent on the postage when 8^{th} set of letters is mailed?

(a) ₹46,930 (b) ₹54,930 (c) ₹87,380 (d) ₹43,690

v. Find the amount spent on the mailing of 9^{th} set?

(a) ₹1,74,762 (b) ₹1,31,072

(c) ₹1,54,536 (d) None of these

Multiple Choice Questions

66. The interior angles of a polygon are in A.P. If the smallest angle be 120° and the common difference be 5, then the number of side is -

(a) 8 (b) 10

(c) 9 (d) 6.

67. α and β are the roots of the equation $x^2 - 3x + a = 0$ and γ and δ are the roots of the equation $x^2 - 12x + b = 0$. If α, β, γ and δ form an increasing G.P., then (a, b)-

(a) (3, 12) (b) (12, 3)

(c) (2, 32) (d) (4, 16).

68. If A be the arithmetic mean between two numbers and S be the sum of n arithmetic means between the same numbers, then -

(a) S = nA (b) A = nS

(c) A = S (d) None of these.

69. If n geometric means be inserted between a and b, then the n^{th} geometric mean will be-

(a) $a\left[\frac{b}{a}\right]^{\frac{n}{n-1}}$ (b) $a\left[\frac{b}{a}\right]^{\frac{n-1}{n}}$

(c) $a\left[\frac{b}{a}\right]^{\frac{n}{n+1}}$ (d) $a\left[\frac{b}{a}\right]^{\frac{1}{n}}$.

70. If the arithmetic and geometric means of two numbers are 10 and 8 respectively, then one number exceeds the other number by-

(a) 8 (b) 10

(c) 12 (d) 16.

71. The first and last terms of A.P. are 1 and 11. If the sum of its term is 36, then the number of terms will be-

(a) 5 (b) 6

(c) 7 (d) 8.

72. If the first, second and last term of an A.P. are a, b and 2a respectively, then its sum is -

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) None of these.

73. If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of this G.P.is -

(a) $\frac{p-q}{q-r}$ (b) $\frac{q-r}{p-q}$

(c) pqr (d) None of these.

74. If A be one A.M. and p, q be two GM's between two numbers, then 2A is equal to-

(a) $\frac{p^3+q^3}{pq}$ (b) $\frac{p^3-q^3}{pq}$

(c) $\frac{p^2+q^2}{2}$ (d) $\frac{pq}{2}$.

75 In a G.P. if the $(m + n)^{th}$ term is p and $(m - n)^{th}$ term is q, then its m^{th} term is -

(a) O (b) pq

(c) $\sqrt{pq}$ (d) $\frac{1}{2}(p+q)$.

ANSWERS

1. 293 2. 11

3. 20^{th} 4. 0

5. $4n + 5$

6. 12th

7. $\frac{9}{2}$

8. 64

9. 3

10. $20\left(1-\frac{1}{2^8}\right)$

11. 5

12. $\frac{1}{3}$

13. $\frac{2}{3}$

14. 89

15. 6

16. $-(p + q)$

17. 15/2

18. 1

19. 4 and 16

20. 3/4

21. $\frac{2n+1}{(n+6)(n+10)^2}$

22. 9

23. $3n + 5$

27. $n = 7$

28. $-\frac{1}{4}$

29. 102, 367

30. 33

31. 7999

32. 952

33. $\frac{3}{5}$

34. 11

35. 36

36. 33:17

37. $\frac{(b+c-2a)(a+c)}{2(b-a)}$

39. 5, 10, 20, ...; or 20, 10, 5,

40. 18, 6, 2; or 2, 6, 18

41. 6

42. $\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$

43. $\frac{7}{81}\left[9n-1+10^{-n}\right]$

45. $\frac{15}{7}\left(2^{n}-1\right)$

51. 1

52. 16384

56. 450 cm²

58. 16, 4

60. 19, $\frac{38}{3}$, $\frac{76}{9}$,......

61. $n + 2^{-n} - 1$

62. $r = 2 + \sqrt{3}$

63. 1, 3, 9

64. i. (a) ii. (c) iii. (b) iv. (c) v. (b)

65. i. (b) ii. (c) iii. (a) iv. (d) v. (b)

66. (c)

67. (c)

68. (a)

69. (c)

70. (c)

71. (b)

72. (c)

73. (b)

74. (a)

75. (c)

CHAPTER - 10

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

  $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

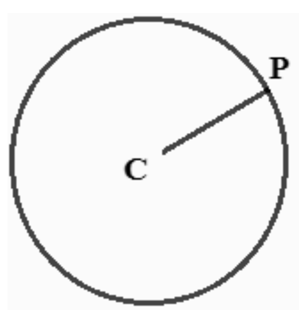
- Let the vertices of a triangle ABC are $A(x_1, y_1)$ $B(x_2, y_2)$ and $C(x_3, y_3)$.Then area of triangle

  $$ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

  Note: Area of a triangle is always positive. If the above expression is zero, then a triangleis not possible. Thus the points are collinear.

- **LOCUS:** When a variable point P(x,y) moves under certain condition then the path traced out by the point P is called the locus of the point.

  For example: Locus of a point P, which moves such that its distance from a fixed point C is always constant, is a circle.

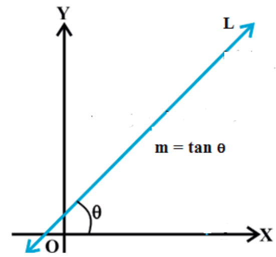


CP = constant

- A line is also defined as the locus of a point satisfying the condition $ax + by + c = 0$ where a, b, c are constants.

- **Slope of a straight line:**

  If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m



  $m = \tan\theta,\ \theta \neq 90°$

  If $0° < \theta < 90°$ then $m > 0$ and

  $90° < \theta < 180°$ then $m < 0$

  Note-1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined

  Note-2: Slope of any horizontal line i.e.|| to x-axis is zero.Slope of a vertical line i.e.|| to y-axis is not defined.

- Three points A, B and C lying in a plane are collinear, if slope of AB = Slope of BC.

- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

- Interecept: There are two types of intercepts x-intercept and y-intercept. The x-intercept is the x-coordinate of the point where line cut x axis while y-intercept is the y coordinate of the point where line cut y axis.

- Two lines are parallel to each other if and only if their slopes are equal.

  i.e., $l_1 \| l_2 \Leftrightarrow m_1 = m_2$.

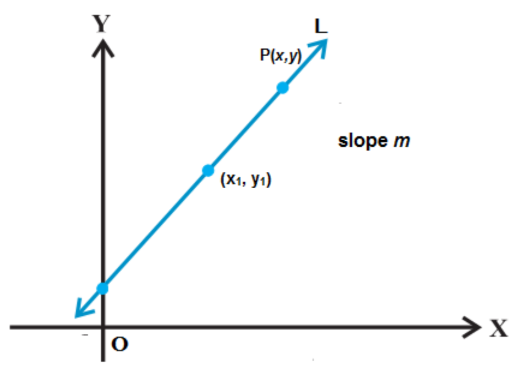
- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

  i.e., $l_1 \perp l_2 \Leftrightarrow m_1 m_2 = -1 \Leftrightarrow m_2 = \dfrac{-1}{m}$.

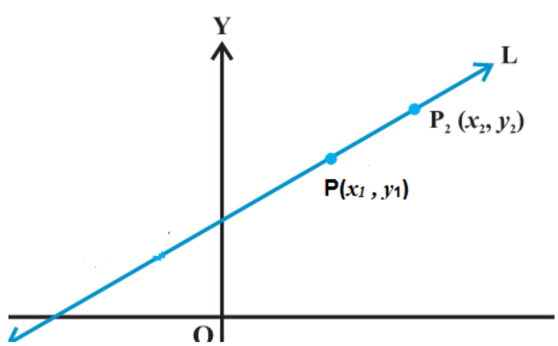
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$ and obtuse angle is $\phi = 180 - \alpha$.

- **Point slope form:**

  Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$



- **Two Point Form:**



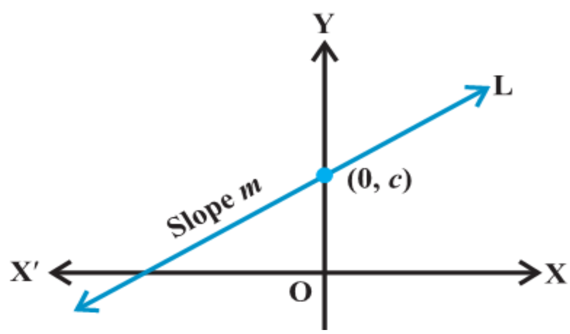
Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

- **Slope intercept form(y-intercept):**

  Equation of a line having slope m and y-intercept 'c' is given by

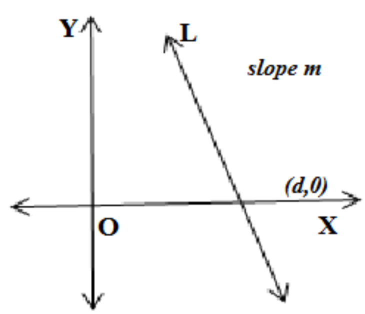
  $y = mx + c$



- **Slope intercept form (x-intercept):**

  Equation of a line having slope m and y-intercept c is given by

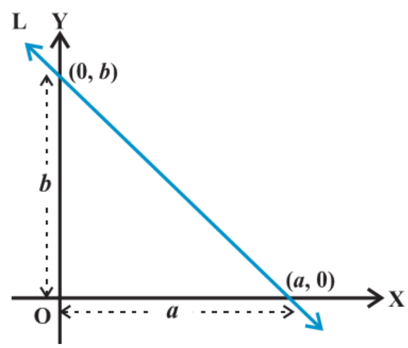
  $y = m (x–d)$



- **Intercept Form:**

  Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

  $$\frac{x}{a}+\frac{y}{b}=1$$



- **General Equation of a line:**

  Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non-zero.

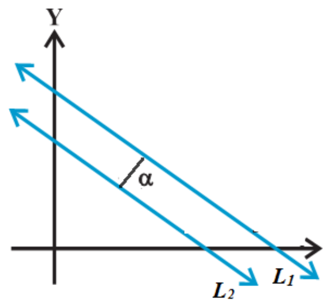
  Slope = $\frac{-A}{B}$ and y-intercept = $\frac{-C}{B}$ x-intercept = $\frac{-C}{A}$.

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

  $$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

  $$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$



VERY SHORT ANSWER TYPE QUESTIONS

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.

2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?

3. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6, q). Find p and q.

4. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1)?

5. Show that the points (a, 0), (0, b) and (3a,– 2b) are collinear.

6. Find the equation of straight line cutting off an intercept –1 from *y* axis and being equally inclined to the axes.

7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

8. Find k so that the line $2x + ky – 9 = 0$ may be perpendicular to$2x + 3y – 1 = 0$

9. Find the acute angle between lines $x + y = 0$ and $y = 0$

10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

11. Find the equation of a line with slope 1/2 and making an intercept 5 on y-axis.

12. Find Equation of line which is parallel to y-axis and at distance 5 units from y-axis.

13. Find the length of perpendicular from a point (1, 2) to a line 3x + 4y + 5 = 0.

SHORT ANSWER TYPE QUESTIONS

14. Determine the equation of line through a point (–4, –3) and parallel to *x*-axis.

15. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices a triangle or not?

16. If a vertex of a triangle is (1, 1) and the midpoints of two sides through this vertex are (–1, 2) and (3, 2). Then find the centroid of the triangle.

17. If the medians through A and B of the triangle with vertices A(0, b), B(0, 0) and C(a, 0) are mutually perpendicular. Then show that $a^2 = 2b^2$.

18. If the image of the point (3, 8) in the line px + 3y – 7 = 0 is the point (–1, –4), then find the value of p.

19. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines $x + 2y = 5$ and $x - 3y + 5 = 0$

20. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are $(-3, 1)$ find coordinates of B.

21. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

22. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

23. Find the equation of a line with slope -1 and whose perpendicular distance from the origin is equal to 5.

24. If a vertex of a square is at $(1, -1)$ and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

25. What is the value of y so that line through (3, y) and (2, 7) is parallel to the line through $(-1, 4)$ and (0, 6)?

26. In what ratio, the line joining $(-1, 1)$ and (5, 7) is divided by the line $x + y = 4$?

27. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and -6 respectively.

28. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.

29. Find the coordinates of the orthocentre of a triangle whose vertices are $(-1, 3)$ $(2, -1)$ and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

30. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

31. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

32. The vertices of a triangle are (6,0), (0,6) and (6,6). Find the distance between its circumcenter and centroid.

LONG ANSWER TYPE QUESTIONS

33. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

34. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

35. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

36. Find the equations of the straight lines which cut off intercepts on x-axis twice that on y-axis and are at a unit distance from origin.

37. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

38. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

39. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

40. If the slope of a line passing through to point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

41. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x - y - 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

42. Find points on the line $x + y + 3 = 0$ that are at a distance of 5 unitsfrom the line $x + 2y + 2 = 0$

43. A straight line L is perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line L and the coordinate axes is 5. Find the equation of the line L.

44. Two equal sides of an isosceles triangle are given by the equation $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side pass through the point (1,–10). Determine the equation of the third side.

45. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2 AC. If the coordinates of D and M are (1,1) and (2,–1) respectively. Then find the coordinates of A.

46. Find the area enclosed within the curve $|x| + |y| = 1$.

47. Find the coordinators of the circumcentre of the triangle whose vertices are (5,7), (6,6) and (2, -2).

48. Find the equation of a straight line, which passes through the point (a, 0) and whose $\perp$ distance from the point (2a, 2a) is a.

49. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2} + b^{-2} = p^{-2} + q^{-2}$.

CASE STUDY TYPE QUESTIONS

50. A person is standing at a point A of a triangular park ABC whose vertices are A(2, 0), B(3, 4) and C(5, 6).

Based on the above information answer the following :-

i. He wants to reach BC in least time. Find the equation of the path he should follow.

(a) $2x + y = 3$ (b) $2x + 3y = 4$

(c) $x + y = 2$ (d) $x + 4y = 7$

ii. Find the shortest distance travelled by him to reach BC -

(a) $\frac{5}{2}\sqrt{2}$ units (b) $\frac{3}{2}\sqrt{2}$ units

(c) $\frac{4}{3}\sqrt{2}$ units (d) $\frac{7}{3}\sqrt{2}$ units

iii. Suppose he meets BC at a point D. Find the coordiantors of the point D.

(a) $\left(\frac{5}{2}, \frac{7}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$

(c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{7}{2}, \frac{5}{2}\right)$

iv. Find the area of the triangular park ABC.

(a) 5 sq units (b) 10 sq units

(c) 3 sq units (d) None of these

v. Find the coordinator of the centroid of the triangular park ABC?

(a) $\left(\frac{5}{3}, \frac{7}{3}\right)$ (b) $\left(\frac{10}{3}, \frac{10}{3}\right)$

(c) $\left(\frac{7}{3}, \frac{8}{3}\right)$ (d) $\left(\frac{2}{3}, \frac{8}{3}\right)$

Multiple Choice Questions

51. The angle between the straight lines $x - y\sqrt{3} = 5$ and $\sqrt{3}x + y = 7$ is-

(a) 90° (b) 60°

(c) 75° (d) 30°

52. If p is the length of the perpendicular drawn from the origin to the line $\frac{x}{a} + \frac{y}{b} = 1$, then which one of the following is correct?

(a) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (b) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$

(c) $\frac{1}{p} = \frac{1}{a} + \frac{1}{b}$ (d) $\frac{1}{p} = \frac{1}{a} - \frac{1}{b}$.

53. What is the equation of the line passing through (2, –3) and parallel to y-axis?

(a) y = –3 (b) y = 2

(c) x = 2 (d) x = –3

54. If the lines $3x + 4y + 1 = 0$, $5x + \lambda y + 3 = 0$ and $2x + y - 1 = 0$ are concurrent, then λ is equal to -

(a) -8 (b) 8

(c) 4 (d) -4.

55. The x-intercept and the y-intercept of the line $5x - 7 = 6y$, respectively are –

(a) $\frac{7}{5}$ and $\frac{7}{6}$ (b) $\frac{7}{5}$ and $\frac{-7}{6}$

(c) $\frac{5}{7}$ and $\frac{6}{7}$ (d) $\frac{-5}{7}$ and $\frac{6}{7}$.

56. If p be the length of the perpendicular from the origin on the straight line $x + 2y = 2q$, then what is the value of q?

(a) $1/p$ (b) p

(c) $p/2$ (d) $\frac{\sqrt{5}p}{2}$

57. A straight line through P(1, 2) is such that its intercept between the axes is bisected at P. Its equation is-

(a) $x + y = -1$ (b) $x + y = 3$

(c) $x + 2y = 5$ (d) $2x + y = 4$.

58. If the lines $3y + 4x = 1$, $y = x + 5$ and $5y + bx = 3$ are concurrent, then what is the value of b?

(a) 1 (b) 3

(c) 6 (d) 0.

59. The triangle formed by the lines $x + y = 0$, $3x + y = 4$ and $x + 3y = 4$ is -

(a) Isosceles (b) Equilateral

(c) Right angled (d) None of these.

60. What is the foot of the perpendicular from the point (2, 3) on the line $x + y - 11 = 0$?

(a) (1, 10) (b) (5, 6)

(c) (6, 5) (d) (7, 4).

ANSWERS

1. (1, 2)
2. $k = 3$
3. $p = 2, q = -5$
4. 3 : 2 (internally)
6. $y = x - 1$ and $y = -x - 1$.
7. $x + y = 7$
8. $\frac{4}{3}$
9. $\frac{\pi}{4}$
10. $\frac{2\pi}{3}$
11. $y = \frac{x}{2} + 5$
12. $x = 5$
13. 16/5
14. $y + 3 = 0$
15. No
16. $\left(1, \frac{7}{3}\right)$

18. 1

19. $\frac{10}{\sqrt{26}}$

20. (1, –5)

21. c = –4, (2,0), (4, 4)

22. 49 square units

23. $x + y + 5\sqrt{2} = 0, \; x + y - 5\sqrt{2} = 0$

24. 4 square units

25. y = 9

26. 1 : 2

27. $2x - 3y - 6 = 0$ and $-3x + 2y - 6 = 0$

28. 4 square units

29. (–4, –3)

30. $x + 2y = 1$

31. $x + y - 5 = 0$

32. $3\sqrt{2}$

33. $3x - 4y + 15 = 0$

34. $4x + 7y - 11 = 0, 7x - 4y + 25 = 0$

    $7x - 4y - 3 = 0$

35. $x - 2y + 3 = 0$, $2x + y - 14 = 0$,

$x - 2y + 13 = 0$, $2x + y - 4 = 0$

$3x - y - 1 = 0$, $x + 3y - 17 = 0$

36. $x + 2y + \sqrt{5} = 0$, $x + 2y - \sqrt{5} = 0$

37. $x = y$

38. $107x - 3y - 92 = 0$

39. $23x - 7y - 9 = 0$ and $7x + 23y - 53 = 0$

40. $(-1, -1)$ or $(7, 5)$

41. $x - 4y - 8 = 0$ or $x + 4y + 8 = 0$

42. $(1,-4)$, $(-9,6)$

43. $x + 5y = \pm 5\sqrt{2}$

44. $x - 3y - 31 = 0$, $3x + y + 7 = 0$

45. $\left(1, \frac{-3}{2}\right)$ or $\left(3, \frac{-1}{2}\right)$

46. $\sqrt{3}$

47. $(2, 3)$

48. $3x - 4y - 3a = 0$ and $x - a = 0$

50. i. (c) ii. (b) iii. (b) iv. (c) v. (b)

51. (a)

52. (a)

53. (c)

54. (b)

55. (b)

56. (b)

57. (d)

58. (c)

59. (a)

60. (b)

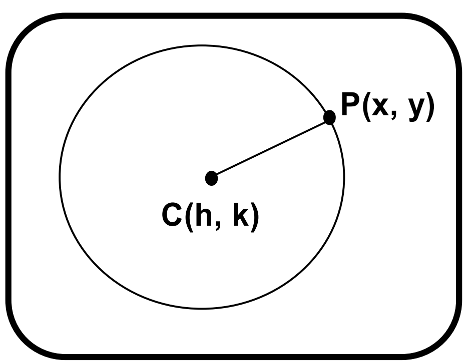
CHAPTER - 11

CONIC SECTIONS

KEY POINTS

- The curves obtained by slicing the cone with a plane not passing through the vertex are called conic sections or simply conics.
- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions.
- A conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed straight line.
- The fixed point is called focus, the fixed straight line is called directrix, and the constant ratio is called eccentricity, which is denoted by '*e*'.
- **Circle:** It is the set of all points in a plane that are equidistant from a fixed point in that plane

  Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$ where Centre (h, k), radius = r



C (h, k)

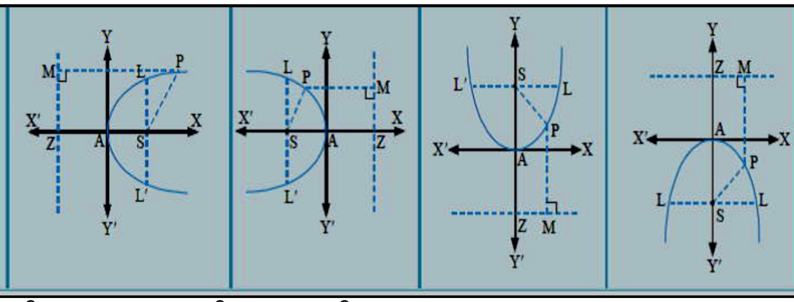
CP = CONSTANT = r

❖ **Parabola:** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line

	$y^2 = 4ax$ Parabola towards right	$y^2 = -4ax$ Parabola towards left	$x^2 = 4ay$ Parabola opening upwards	$x^2 = -4ay$ Parabola opening downwards
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$

Note: In the standard equation of parabola, $a > 0$.

Figure

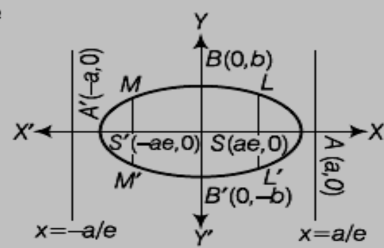
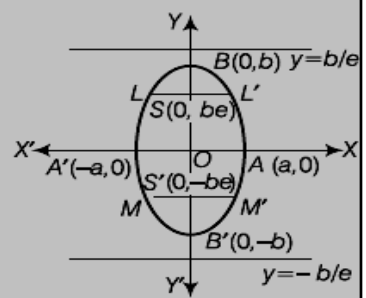


$y^2 = 4ax$ $y^2 = -4ax$ $x^2 = 4ay$ $x^2 = -4ay$

Note: In the figure above, A represents the vertex, S represents the Focus, LL' represents the Latus Rectum and Line MZ represents the Directrix to the parabola.

- **Latus Rectum:** A chord through focus perpendicular to axis of parabola is called its latus rectum.

- **Ellipse:** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points.

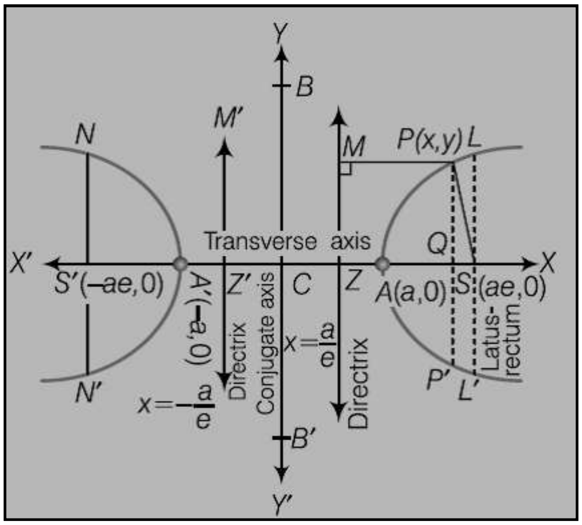
Standard equation	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\,(a>b)$ (Horizontal form of an ellipse)	$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\ (a<b)$ (Vertical form of an ellipse)
Shape of the ellipse		
Centre	(0, 0)	(0, 0)
Equation of major axis	$y = 0$	$x = 0$
Equation of minor axis	$x = 0$	$y = 0$
Length of major axis	$2a$	$2b$
Length of minor axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Equation of directrices	$x = \pm \dfrac{a}{e}$	$y = \pm \dfrac{b}{e}$
Eccentricity	$e = \sqrt{\dfrac{a^2 - b^2}{a^2}}$	$e = \sqrt{\dfrac{b^2 - a^2}{b^2}}$
Length of latusrectum	$\dfrac{2b^2}{a}$	$\dfrac{2a^2}{b}$

Note: If e = 0 for an ellipse then b = a and equation of ellipse will be converted in equation of the circle. Its eq. will be $x^2 + y^2 = a^2$. It is called auxiliary circle. For auxiliary circle, diameter is equal to length of major axis and e = 0.

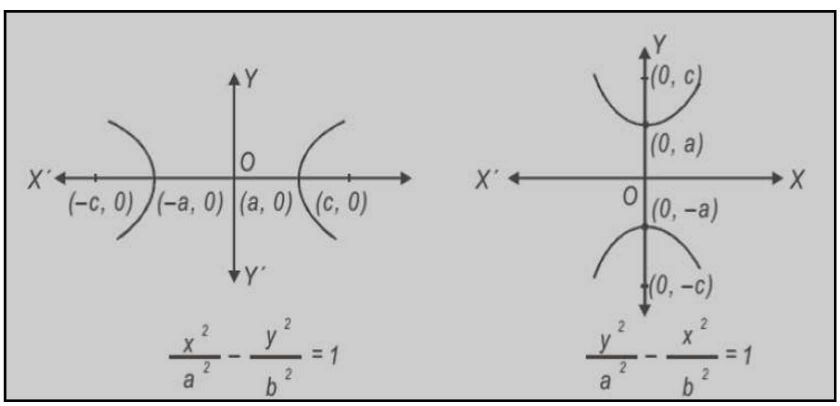
- **Latus rectum:** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola:** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.

	Hyperbola	**Conjugate hyperbola**
Standard equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$
Centre	(0, 0)	(0, 0)
Equation of transverse axis	$y = 0$	$x = 0$
Equation of conjugate axis	$x = 0$	$y = 0$
Length of transverse axis	$2a$	$2b$
Length of conjugate axis	$2b$	$2a$
Foci	$(\pm ae, 0)$	$(0, \pm be)$
Equation of directrices	$x = \pm \frac{a}{e}$	$y = \pm \frac{b}{e}$
Vertices	$(\pm a, 0)$	$(0, \pm b)$
Eccentricity	$e = \sqrt{\frac{a^2 + b^2}{a^2}}$	$e = \sqrt{\frac{a^2 + b^2}{b^2}}$
Length of latusrectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$

- **STANDARD HYPERBOLA:**



- **STANDARD HYPERBOLACONJUGATE HYPERBOLA:**



- **Latus Rectum:** Chord through foci perpendicular to transverse axis is called latus rectum.

  If $e = \sqrt{2}$ for hyperbola, then hyperbola is called rectangular hyperbola.

  For $e = \sqrt{2}$ then b = a and eq. of its hyperbola will be $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$.

VERY SHORT ANSWER TYPE QUESTIONS

1. Find the centre of the circle $3x^2 + 3y^2 + 6x - 12y - 6 = 0$.

2. Find the radius of the circle $3x^2 + 3y^2 + 6x - 12y - 15 = 0$.

3. Find the equation of circle whose end points of one of its diameter are $(-2, 3)$ and $(0, -1)$.

4. If parabola $y^2 = px$ passes through point $(2, -3)$, then, find the length of latus rectum.

5. Find the coordinates of focus of parabola $3y^2 = 8x$.

6. Find the equation of the circle which passes through the point $(4, 6)$ and has its centre at $(1, 2)$.

7. Find the equation of the ellipse having foci $(0, 3)$, $(0, -3)$ and minor axis of length 8.

8. Find the length of the latus rectum of the ellipse $3x^2 + y^2 = 12$.

9. Find the eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half of the distance between the foci.

10. If the lines $5x + 12y = 3$ and $10x + 24y - 58 = 0$ are tangents to a circle, then find the radius of the circle.

11. Find the length of major and minor axis of the following ellipse, $16x^2 + 25y^2 = 400$.

12. Find the eqn. of hyperbola satisfying given conditions foci $(\pm 5, 0)$ and transverse axis is of length 8.

13. Find the coordinates of points on parabola $y^2 = 8x$ whose focal distance is 4.

14. Find the distance between the directrices to the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$.

15. If the eccentricity of the ellipse is zero. Then show that ellipse will be a circle.

16. If the eccentricity of the hyperbola is $\sqrt{2}$. Then find the general equation of hyperbola.

17. A circle is circumscribed on an equilateral Triangle ABC where AB = 6 cm. The area of the Circumcircle is $K\pi$ cm^2. Find the value of K.

SHORT ANSWER TYPE QUESTIONS

18. Find equation of an ellipse having vertices (0, ± 5) and foci (0, ± 4).

19. If the distance between the foci of a hyperbola is 16 and its eccentricity is 2, then obtain the equation of a hyperbola.

20. Find the equation for the ellipse that satisfies the given condition Major axis on the *x*-axis and passes through the points (4, 3) and (6, 2).

21. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of diameter.

22. Find the equation of the ellipse with foci at (± 5, 0) and x = 1.8 as one of the directrices.

23. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, find the equation of the hyperbola if its eccentricity is 2.

24. Find the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which passes through the points (3, 0) and ($3\sqrt{2}$, 2).

25. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

26. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

27. Find the equation of a circle whose centre is at (4, –2) and $3x - 4y + 5 = 0$ is tangent to circle.

28. If equation of the circle is in the form of $x^2 + y^2 + 2gx + 2fy + c = 0$ then prove that its centre and radius will be (–g, –f) and $\sqrt{g^2+f^2-c}$ respectively.

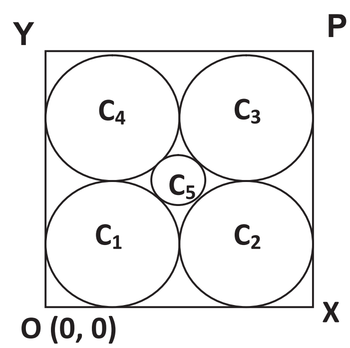
29. If the end points of a diameter of circle are (x_1, y_1) and (x_2, y_2) then show that equation of circle will be $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

30. Find the equation of the circle which touches the lines x = 0, y = 0 and x = 2c and c > 0.

31. Find the equation of the set of all points the sum of whose distance from A(3,0) and B(9,0) is 12 unit. Identify the curve thus obtained.

32. Find the equation of the set of all points such that the difference of their distance from (4,0) and (–4,0) is always equal of 2 unit. Identify the curve thus obtained.

33. If OXPY is a square of Side 4 cm in First Quadrant, where O is the origin. (OY and OX are lies y-axis and x-axis respectively). Find the equation of the circle C_1, C_2, C_3, $C_4$and C_5.



LONG ANSWER TYPE QUESTIONS

34. Prove that the points (1, 2), (3, – 4), (5, – 6) and (11, – 8) are concyclic.

35. A circle has radius 3 units and its centre lies on the line y = x – 1. If it is passes through the point (7, 3) then find the equations of the circle.

36. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, – 9). Find its centre and radius.

37. Find the equation of circle having centre (1, – 2) and passing through the point of intersection of the lines 3x + y = 14 and 2x + 5y = 18.

38. Show that the points A(5,5), B(6,4), C(–2,4) and D(7,1) all lies on the circle. Find the centre, radius and equation of circle.

39. Find the equation of the ellipse in which length of minor axis is equal to distance between foci. If length of latus rectum is 10 unit and major axis is along the x axis.

40. Find the equation of the hyperbolas whose axes (transverse and conjugate axis) are parallel to x axis and y axis and centre is origin such that Length of latus rectum length is 18 unit and distance between foci is 12 unit.

41. Prove that the line $3x + 4y + 7 = 0$ touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$. Also find the point of contact.

42. Find the equations of tangents to the circle

(a) $x^2 + y^2 - 2x - 4y - 4 = 0$ which are parallel to $3x - 4y - 1 = 0$

(b) $x^2 + y^2 - 4x - 6y - 12 = 0$ which are perpendicular to $4x + 3y = 7$

43. Find the equation of Circle in each of the following cases:

(a) Touches both the coordinate axes in first quadrant and having radius = 1 unit

(b) Touches both the coordinate axes in second quadrant and having radius = 2 units

(c) Touches both the coordinate axes in third quadrant and having radius = 3 units

(d) Touches both the coordinate axes in fourth quadrant and having radius = 4 units

(e) Touches the x-axis at origin and having radius = 5 units

(f) Touches the y-axis at origin and having radius = 6 units

CASE STUDY TYPE QUESTIONS

44. A beam is supported at its ends by supports which are 12m apart. Since the load is concentrated at its centre, there is a

deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola.

Based on the above information answer the following :-

i. 1How for from the centre is deflection of 1cm?

(a) $2\sqrt{6}$ m (b) $3\sqrt{6}$ m

(c) $2\sqrt{3}$ m (d) $4\sqrt{3}$ m

ii. What will be the equation of parabola?

(a) $x^2 = 2400y$ (b) $x^2 = 1200y$

(c) $x^2 = 1600y$ (d) $x^2 = 1000y$

iii. At a distance of 2m from the centre, what will be the deflection of the beam?

(a) $\frac{2}{300}$ (b) $\frac{8}{300}$

(c) $\frac{3}{400}$ (d) $\frac{1}{500}$

iv. What is the length of latus rectum of the parabola?

(a) 1000 (b) 1200

(c) 1300 (d) 1400

v. What is the difference of deflection of beam at a distance of 1m and 2m from the centre?

(a) $\frac{1}{300}$ (b) $\frac{1}{500}$

(c) $\frac{1}{400}$ (d) $\frac{3}{700}$

45. A window is in the shape of parabola with a triangle inscribed in it. The triangle is formed in such a way that the verticles of triangle coincides with vertex of parabola and end points of latus rectum. The equation of parabola is given by $x^2 – 24y$.

Based on the above information answer the following :-

i. Which type of parabola is represented by the given equation?

(a) Parabola towards right

(b) Parabola towards left

(c) Parabola opening upwards

(d) Parabola opening downwards

ii. Find the length of altitude of the triangle -

(a) 12 units (b) 6 units

(c) 18 units (d) 3 units

iii. Find the area of the triangle?

(a) 60 sq. units (b) 96 sq. units

(c) 72 sq. units (d) 110 sq. units

iv. Find the length of the longest side of the triangle?

(a) $6\sqrt{5}$ units (b) 24 units

(c) 12 units (d) 48 units

v. Find the length of latus rectum of the parabola?

(a) 24 units (b) 12 units

(c) 6 units (d) 48 units

Multiple Choice Questions

46. The equation of the circle which passes through the points of intersection of the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 - 6y = 0$ and has its centre at (3/2, 3/2) is -

    (a) $x^2 + y^2 + 3x + 3y + 9 = 0$

    (b) $x^2 + y^2 + 3x + 3y = 0$

    (c) $x^2 + y^2 - 3x - 3y = 0$

    (d) $x^2 + y^2 - 3x - 3y + 9 = 0$.

47. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ -

    (a) (4,9) (b) (9,4)

    (c) (7,4) (d) (4,7).

48. Value of p, for which the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ represent a circle of radius 5 units is -

    (a) 3 (b) – 3

    (c) both (a) & (b) (d) Neither (a) nor (b).

49. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is 'e' then the value of '5e' is -

    (a) 3 (b) 4

    (c) 2 (d) 1.

50. The centre of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is (a, b) then (2a + 3 b) is -

    (a) 0 (b) 2

    (c) 3 (d) 5.

51. The radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is -

(a) 1 (b) 2

(c) 3 (d) 5.

52. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 8y$ to the ends of its latus rectum is -

(a) 4 sq. units (b) 8 sq. units

(c) 12 sq. units (d) 16 sq. units.

53. Match the following:

	COLUMN 1 Conic		**COLUMN 2 Eccentricity**
A	CIRCLE	P	$e < 1$
B	PARABOLA	Q	$e > 1$
C	ELLIPSE	R	$e = 0$
D	HYPERBOLA	S	$e = 1$

Which one of the following is true?

$A \rightarrow P$, $B \rightarrow Q$, $C \rightarrow R$, $D \rightarrow S$

$A \rightarrow S$, $B \rightarrow Q$, $C \rightarrow R$, $D \rightarrow P$

$A \rightarrow Q$, $B \rightarrow S$, $C \rightarrow R$, $D \rightarrow P$

$A \rightarrow R$, $B \rightarrow S$, $C \rightarrow P$, $D \rightarrow Q$

54. At what point on the parabola $x^2 = 9y$ is the abscissa three times that of ordinate

(a) (1, 1) (b) (3, 1)

(c) (–3, 1) (d) (–3, –3)

55. The equation of parabola with vertex at origin and axis on x-axis and passing through point (2, 3) is

(a) $y^2 = 9x$ (b) $y^2 = \frac{9x}{2}$

(c) $y^2 = 2x$ (d) $y^2 = \frac{2x}{9}$

ANSWERS

1. $(-2, 4)$
2. 5 Units
3. $x^2 + y + 2x - 2y - 3 = 0$
4. 4.5 units
5. $\left(\frac{2}{3}, 0\right)$
6. $(x - 4)^2 + (y - 2)^2 = 25$
7. $\frac{x^2}{16} + \frac{y^2}{25} = 1$
8. $\frac{4\sqrt{3}}{3}$
9. $e = 2$
10. 2 units
11. Length of Major Axis = 10
    Length of Major Axis = 8
12. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

13. (2, ±4)

14. 18

16. $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$

17. K = 12

18. $\frac{x^2}{9}+\frac{y^2}{25}=1$

19. $x^2 - y^2 = 32$

20. $\frac{x^2}{52}+\frac{y^2}{13}=1$

21. (1, 2)

22. $\frac{x^2}{36}+\frac{y^2}{11}=1$

23. $\frac{x^2}{4}-\frac{y^2}{12}=1$

24. $e=\frac{\sqrt{13}}{3}$

25. $e=\frac{\sqrt{3}}{2}$

26. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

27. $x^2 + y^2 - 8x + 4y - 5 = 0$

30. $x^2 + y^2 - 2cx \pm 2cy + c^2 = 0$

31. $3x^2 + 4y^2 = 36$, Ellipse

32. $15x^2 - y^2 = 15$, Hyperbola

33. $C_1: (x-1)^2 + (y-1)^2 = 1$

$C_2: (x-3)^2 + (y-1)^2 = 1$

$C_3: (x-3)^2 + (y-3)^2 = 1$

$C_4: (x-1)^2 + (y-3)^2 = 1$

$C_5: (x-2)^2 + (y-2)^2 = \left(\sqrt{2}-1\right)^2$

35. $x^2 + y^2 - 8x - 6y + 16 = 0$ or

$x^2 + y^2 - 14x - 12y + 76 = 0$

36. $x^2 + y^2 - 14x - 6y - 111 = 0$

Centre (7, 3), Radius = 13 units

37. $(x-1)^2 + (y+2)^2 = 25$

38. $x^2 + y^2 - 4x - 2y - 20 = 0$

Centre(2, 1), Radius = 5 units

39. $x^2 + 2y^2 = 100$

40. $3x^2 - y^2 = 27$

41. Point of contact = $(-1, -1)$

42. (a) $3x - 4y - 10 = 0$ or $3x - 4y + 20 = 0$

(b) $3x - 4y + 31 = 0$ or $3x - 4y - 19 = 0$

43. (a) $(x-1)^2 + (y-1)^2 = 1$ (b) $(x+2)^2 + (y-2)^2 = 4$

(c) $(x+3)^2 + (y+3)^2 = 9$ (d) $(x-4)^2 + (y+4)^2 = 16$

(e) $x^2 + (y \pm 5)^2 = 25$ (f) $(x \pm 6)^2 + y^2 = 36$

44. i. (a) ii. (b) iii. (b) iv. (b) v. (c)

45. i. (d) ii. (b) iii. (c) iv. (b) v. (a)

46. (c)

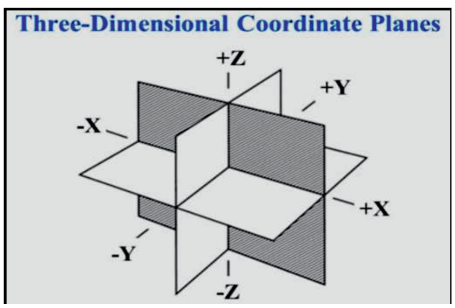
47. (d)
48. (c)
49. (b)
50. (a)
51. (d)
52. (b)
53. (d)
54. (b)
55. (b)

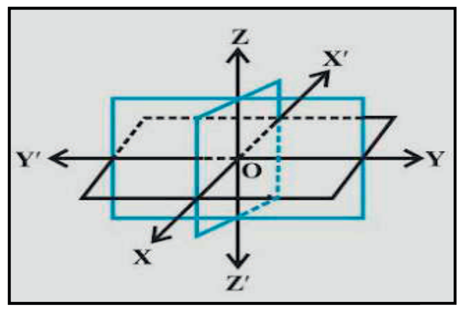
CHAPTER - 12

INTRODUCTION TO THREE-DIMENSIONAL COORDINATE GEOMETRY

KEY POINTS

- Three mutually perpendicular lines X′OX, Y′OY and Z′OZ in space **constitute rectangular coordinate system** which in turn divide the space into eight parts known as **octants** and the lines are known as **Coordinate axes**.





❖ **Coordinate axes:** XOX', YOY', ZOZ' are respectively called x-axis, y-axis and z-axis.

❖ **Coordinate planes:** XOY, YOZ, ZOX or XY, YX, ZX planes

❖ **Octants:** XOYZ, X'OYZ, X'OY′Z, XOY'Z, XOYZ', X'OYZ′, X′OY'Z' and XOY′Z' denoted as I, II, ….. VIII octant respectively.

❖ Coordinates of a points lying on x-axis, y-axis and z-axis are of the form (x,0,0), (0,y,0),(0,0,z) respectively.

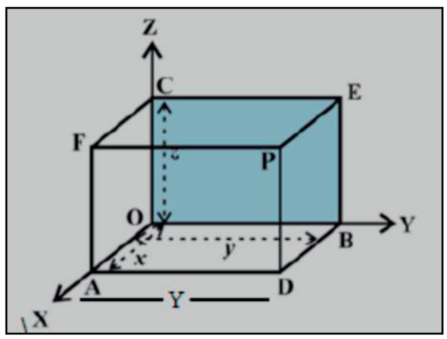
❖ The signs of coordinates in eight octants are as follows:

(i) (+ + +)	(iii) (- - +)	(v) (+ + -)	(vii) (- - -)
(ii) (- + +)	(iv) (+ - +)	(vi) (- + -)	(viii) (+ - -)

❖ Coordinates of a points lying on xy-plane, yz-plane and xz-plane are of the form (x,y,0), (0,y,z),(x,0,z) respectively.

❖ The reflection of the point (x, y, z) in xy-plane, yz-plane and xz-plane is (x, y, –z), (–x, y,z) and (x, –y, z) respectively.

❖ Absolute value of the Coordinates of a point P (x, y, z) represents the perpendicular distances of point P from three coordinate planes YZ, ZX and XY respectively.



● The distance between the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

VERY SHORT ANSWER TYPE QUESTIONS

1. What will be the image of (–1, 2, –3) in XZ plane.
2. What will be the image of (–1, 2, –3) in YZ plane.
3. In which octant the Point P (–5, 4, –3) lies?
4. If a < 0, b > 0 & c < 0, in which octant the Point P (a, b, –c) lies.

5. Find the perpendicular distance of the point P(- 6, 7, –8) from xy-plane.

6. Find the perpendicular distance of the point P(- 3, 5, –12) from x-axis.

7. Find the perpendicular distance of the point P(- 3, 4, –5) from z-axis.

8. Find the coordinates of foot of perpendicular from (3, 7, 9) on y-axis.

9. If the distance between the points (a, 2, 1) and (1, –1, 1) is 5, then find the sum of all possible value of a.

10. Name the axis formed by intersection of two planes xy-plane and yz-plane.

11. Find Distance of the point (3, 4, 5) from the origin (0, 0, 0).

12. If $(c - 1) > 0$, $(a + 2) < 0$ and $b > 0$ then the point P(a, –b, c) lies in which octant?

13. What are the coordinates of the vertices of a cube whose edge is 2 unit, one of whose vertices coincides with the origin and the three edges passing through the origin coincides with the positive direction of the axes through the origin?

14. Let A, B, C be the feet of perpendiculars from point P(1, –2, –3) on the xy-plane, yz-plane and xz-plane respectively. Find the coordinates of A, B, C.

15. If a parallelepiped is formed by planes drawn through the point (5,8,10) and (3,6,8) parallel to the coordinates planes, then find the length of the diagonal of the parallelepiped.

16. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13,10 and 8 unit.

17. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.

18. Find the point on y–axis which is equidistant from the point (3,1,2) and (5, 5, 2).

19. Determine the point in yz plane which is equidistant from three points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

20. Find the length of the medians of the triangle with vertices A(0, 0, 3), B(0, 4, 0) and C (5, 0, 0).

21. If the extremities (end points) of a diagonal of a square are (1,– 2, 3) and (2,–3, 5) then find the length of the side of square.

22. Three consecutive vertices of a parallelogram ABCD are A(6,–2,4) B(2, 4,–8), C(–2, 2, 4). Find the coordinates of the fourth vertex.

23. If the points A(1, 0, –6), B(3, p, q) and C(5, 9, 6) are collinear, find the value of p and q.

24. Show that the point A(1, 3, 0), B(–5, 5,, 2), C(–9, –1, 2) and D(–3, –3, 0) are the vertices of a parallelogram ABCD, but it is not a rectangle.

25. Describe the vertices and edges of the rectangular parallelepiped with one vertex (3, 5, 6) placed in the first octant with one vertex at origin and edges of parallelepiped lie along x, y and z-axis.

26. Find the coordinates of the point which is equidistant from the point (3, 2, 2), (–1, 2, 2), (4, 5, 6) and (2, 1, 2).

27. Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) form a right angled isosceles triangle.

28. Show that the points (5, -1, 1), (7, -4, 7), (1, -6, 10) and (-1, -3, 4) are the vertices of a rhombus.

CASE STUDY TYPE QUESTIONS

29. Consider a ΔABC with vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$. AD, BE and CF are medians of ΔABC.

Based on the above information, answer the following questions:-

i. Coordinates of Point D are?

(a) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$ (b) $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$

(c) $\left(\frac{x_3+x_1}{2}, \frac{y_3+y_1}{2}, \frac{z_3+z_1}{2}\right)$ (d) None of these

ii. A point G divides AD in 2 : 1, the coordinates of G are

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}, \frac{z_1+2z_2}{3}\right)$

(c) $\left(\frac{x_2+2x_1}{3}, \frac{y_2+2y_1}{3}, \frac{z_2+2z_1}{3}\right)$

(d) None of these

iii. For ΔABC, G is

(a) Incentre (b) Circumcentre

(c) Centroid (d) Orthocentre

iv. G divides BE in ratio

(a) 1 : 2 (b) 2 : 1

(c) 3 : 1 (d) 1 : 3

v. If ΔABC is equilateral, then coordinates of circumcentre are

(a) $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

(b) $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}, \frac{z_1+z_2+z_3}{2}\right)$

(c) $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$

(d) None of these

30. ABCD is a field in shape of parallelogram coordinate of A, B and C are (3, –1, 2), (1, 2, –4) and (–1, 1, 2) resp.

Based on the above information answer the following :-

i Coordinates of mid point of AC be

(a) (–1, 1, 2) (b) (–1, 0, –2)

(c) (1, 0, 2) (d) (1, 0, –2)

ii. Coordinates of D be

(a) (1, –2, 8) (b) (–1, –2, 8)

(c) (0, –2, 8) (d) (1, 2, 8)

iii. Length of side BC is

(a) 7 (b) $\sqrt{41}$

(c) $\sqrt{47}$ (d) $\sqrt{43}$

iv. Coordinates of centroid G of ΔABC be

(a) $\left(\frac{5}{3}, \frac{1}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{-5}{3}, \frac{-1}{3}, \frac{2}{3}\right)$

(c) $\left(\frac{5}{3}, -1, \frac{2}{3}\right)$ (d) $\left(1, \frac{2}{3}, 0\right)$

v. Length of AC is

(a) $5\sqrt{2}$ (b) $2\sqrt{5}$

(c) $\sqrt{10}$ (d) 20

Multiple Choice Questions

31. A point on ZX-plane which is equidistant from the points (1, -1, 0), (2, 1, 2), (3, 2, -1) is

(a) $\left(\frac{1}{5}, 0, \frac{31}{10}\right)$ (b) $\left(\frac{1}{10}, 0, \frac{31}{5}\right)$

(c) $\left(\frac{31}{10}, 0, \frac{1}{5}\right)$ (d) $\left(\frac{31}{5}, 0, \frac{1}{10}\right)$

32. Lengths of medians of triangle ABC with vertices

A(0, 0, 2), B(0, 4, 0) and C(8, 0, 0) are:

(a) $2\sqrt{6}, \sqrt{33}, \sqrt{69}$ (b) 2, 4, 8

(c) 8, 4, 2 (d) $2\sqrt{5}, 10, 2\sqrt{17}$

33. A point on y-axis which is at a distance of $\sqrt{10}$ from the point (1, 2, 3) is

(a) (2, 0, 2) (b) (0, 2, 2)

(c) (2, 2, 2) (d) (0, 2, 0)

34. The locus of a point for which y = 0, z = 0 is

(a) x-axis (b) y-axis

(c) z-axis (d) y and z-axes

35. A line is parallel to xy-plane if all points on the line have equal

(a) x-coordinates (b) y-coordinates

(c) z-coordinates (d) x and y-coordinate

36. x-axis is the intersection of two planes

(a) xy and xz (b) yz and zx

(c) xy and yz (d) none of these

37. If the distance between the points (a, 0, 1) and (0, 1, 2) is $\sqrt{27}$, then the value of a is

(a) 5 (b) ± 5

(c) – 5 (d) None of these

38. The point (2, 3, –4) lies in the

(a) First octant (b) Second octant

(c) Fifth octant (d) Seventh octant

39. x = a. represents a plane parallel to

(a) xy – plane (b) yz-plane

(c) xz-plane (d) none of these

40. The distance between the point (a, b, c) and (0, 0, -c) is

(a) $\sqrt{a^2 + b^2}$ (b) $\sqrt{a^2 + b^2 + c^2}$

(c) $\sqrt{a^2 + b^2 + 2c^2}$ (d) $\sqrt{a^2 + b^2 + 4c^2}$

ANSWERS

1. (–1, –2, –3)
2. (1, 2, –3)
3. Octant VI
4. Octant II
5. 8 units
6. 3 units
7. 5 units
8. (0, 7, 0)
9. 5 + (–3) = 2
10. Y-axis
11. $5\sqrt{2}$
12. Octant III
13. (2, 0, 0) , (2, 2, 0) , (0, 2, 0) , (0, 2, 2) , (2, 0, 2), (0, 0, 0) , (2, 2, 2)
14. (4, –3, 0) , (0, –3, –5) , (4, 0, –5)
15. $2\sqrt{3}$
16. $\sqrt{333}$

18. (0, 5, 0)

19. (0, 1, 3)

20. $7, \sqrt{34}, 7$

21. $\sqrt{3}$

22. (2, –4, 16)

23. p = 6, q = 2

25. (0, 0, 0), (3, 0, 0), (3, 5, 0), (0, 5, 0), (0, 5, 6)
(0, 0, 6), (3, 0, 6), (3, 5, 6), $\sqrt{61}$, $\sqrt{45}$, $\sqrt{34}$

26. (1, 3, 5)

27. [Hint: length of three side of triangle is $3\sqrt{2}, 3\sqrt{2}, 6$]

28. [Hint: each side= 7 units]

29. i. (b) ii. (a) iii. (c) iv. (b) v. (a)

30. i. (c) ii. (a) iii. (b) iv. (d) v. (b)

31. (c) 32. (a)

33. (d) 34. (a)

35. (c) 36. (a)

37. (b) 38. (c)

39. (b) 40. (d)

CHAPTER - 13

LIMITS AND DERIVATIES

KEY POINTS

- To check whether limit of f(x) as x approaches to a exists i.e., $\lim\limits_{x \to a} f(x)$ exists, we proceed as follows.

  (i) Find L.H.L at x = a using L.H.L. = $\lim\limits_{h \to 0} f(a-h)$.

  (ii) FindR.H.L at x = a using R.H.L. = $\lim\limits_{h \to 0} f(a+h)$.

  (iii) If both L.H.L. and R.H.L. are finite and equal, then limit at x = a i.e., $\lim\limits_{x \to a} f(x)$ exists and equals to the value obtained from L.H.L or R.H.L else we say "limit does not exist".

- $\lim\limits_{x \to a} f(x) = l,$ if and only if $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = l$

▶ **ALGEBRA OF LIMITS:** Let f, g be two functions such that $\lim\limits_{x \to c} f(x) = l, \text{ and } \lim\limits_{x \to c} g(x) = m.$

  - $\lim\limits_{x \to c} [\alpha f(x)] = \alpha \lim\limits_{x \to c} f(x) = \alpha l, \text{ for all } \alpha \,\varepsilon\, R$

  - $\lim\limits_{x \to c} [f(x) \pm g(x)] = \lim\limits_{x \to c} f(x) \pm \lim\limits_{x \to c} g(x) = l \pm m$

  - $\lim\limits_{x \to c} [f(x) \cdot g(x)] = \lim\limits_{x \to c} f(x) \cdot \lim\limits_{x \to c} g(x) = l \cdot m$

  - $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} g(x)} = \dfrac{l}{m}, \ m \neq 0, \ g(x) \neq 0$

- $\lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l},\ l \neq 0, f(x) \neq 0$
- $\lim_{x \to c} [f(x)]^n = [(\lim_{x \to c} f(x)]^n = l^n$, for all n ε N

▶ SOME IMPORTANT RESULTS ON LIMITS:

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{\tan x}{x} = 1$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$
- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$
- $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$
- $\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$
- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x)$

▶ SOME IMPORTANT RESULTS ON DERIVATIVE:

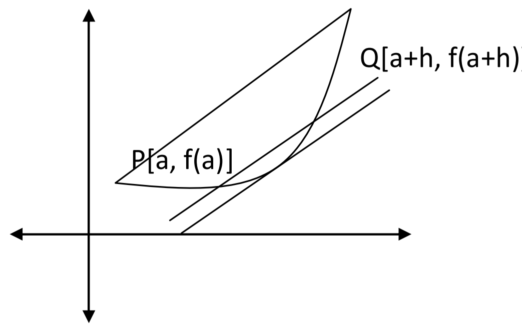
- $\dfrac{d(\sin x)}{dx} = \cos x$
- $\dfrac{d(\cos x)}{dx} = -\sin x$
- $\dfrac{d(\tan x)}{dx} = \sec^2 x$
- $\dfrac{d(\cot x)}{dx} = -\text{cosec}^2 x$
- $\dfrac{d(\sec x)}{dx} = \sec x \cdot \tan x$
- $\dfrac{d(\cos ecx)}{dx} = -\text{cosec}x \cdot \cot x$

- $\dfrac{d(x^n)}{dx} = n \cdot x^{n-1}$
- $\dfrac{d(\sqrt{x})}{dx} = \dfrac{1}{2\sqrt{x}}$
- $\dfrac{d(a)}{dx} = 0,\ a = \text{constan } t$
- $\dfrac{d(e^x)}{dx} = e^x$
- $\dfrac{d(\log x)}{dx} = \dfrac{1}{x}$
- $\dfrac{d(a^x)}{dx} = a^x \cdot \log a$

▶ Logarithm Properties:

- $\log_e(A \cdot B) = \log_e A + \log_e B$
- $\log_e\left(\dfrac{A}{B}\right) = \log_e A - \log_e B$
- $\log_e(A^m) = m \cdot \log_e A$
- $\log_a(1) = 0$
- $\log_B(A) = x,\ \text{then } B^x = A$

- Let y = f(x) be a function defined in some neighbourhood of the point 'a'. Let P[a, f(a)] and Q[a + h, f(a + h)] are two points on the graph of f(x) where h is very small and h ≠ 0.



$$\text{Slope of } PQ = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- If $\lim\limits_{h \to 0}$ point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

  $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ (if exists) is called derivative of f(x) at the point 'a'.

  It is denoted by f'(a).

▶ ALGEBRA OF DERIVATIVES:

- $\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx}[f(x)],\ \text{where } c \text{ is a constan } t$

- $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

  Product Rule:

- $\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot \frac{d}{dx}[g(x)] + g(x) \frac{d}{dx}[f(x)]$

Quotient Rule:

- $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\cdot\frac{d}{dx}[f(x)]-f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h,k) is given by $\frac{dy}{dx}\Big|_{(h,k)}$ and is denoted by 'm'

VERY SHORT ANSWER TYPE QUESTIONS

1. Evaluate $\lim\limits_{x\to 3}\frac{4x+3}{x-2}$

2. Evaluate $\lim\limits_{x\to 2}\frac{x^2+x-2}{x-1}$

3. Evaluate $\lim\limits_{x\to 2}\frac{x^4-16}{x-2}$

4. Evaluate $\lim\limits_{x\to 0}\frac{(1+x)^8-1}{x}$

5. Evaluate $\lim\limits_{x\to 0}\frac{\sin^2 3x}{x^2}$

6. Evaluate $\lim\limits_{x\to 0}\frac{\sin^3 x/2}{x^3}$

7. Evaluate $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$

8. Evaluate $\lim\limits_{x\to 0}\frac{2^x-1}{x}$

9. Evaluate $\lim_{x \to 0} \dfrac{5^x - 1}{3^x - 1}$

10. Evaluate $\lim_{x \to 0} \dfrac{3^{-2x} - 1}{x}$

11. Evaluate $\lim_{x \to 0} \dfrac{\log(1-3x)}{x}$

12. Evaluate $\lim_{x \to 0} \dfrac{7^x - 1}{\tan x}$

13. Differentiate $f(x) = x^2 + \cos x$

14. If $y = (x+1)(x-2)$, find $\dfrac{dy}{dx}$

15. If $y = \dfrac{x^5}{x-3}$, find $\dfrac{dy}{dx}$

Short Answer Type Questions

16. Evaluate $\lim_{x \to 0} \dfrac{(1+x)^m - 1}{(1+x)^n - 1}$

17. Evaluate $\lim_{x \to 0} \dfrac{(\sin 2x) + 3x}{2x + (\tan 3x)}$

18. Evaluate $\lim_{x \to 0} \dfrac{1 - \cos 2x}{1 - \cos 4x}$

19. If y = $\sin^2 x . \cos^3 x$, then $\dfrac{dy}{dx}$.

20. If y = sin 2x.cos3x, then $\dfrac{dy}{dx}$.

21. Differentiate $\frac{\sin x}{x}$ with respect to x.

22. Differentiate $x^3 + 3^3 + 3^x$ with respect to x.

23. Differentiate $\sin^2(x^3+x-1) + \frac{1}{\sec^2(x^3+x-1)}$ with respect to x.

24. Differentiate $\left(\frac{x^a}{x^b}\right)^{a+b} \cdot \left(\frac{x^b}{x^c}\right)^{b+c} \cdot \left(\frac{x^c}{x^a}\right)^{c+a}$ with respect to x.

25. Differentiate $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{a-c}+x^{b-c}}$ w.r.t to x.

26. Find the derivative of x using first principle method.

27. If $\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to k} \frac{x^3-k^3}{x^2-k^2}$, then find the value of k.

28. Find the derivative of $(x - 1)(x + 1)(x^2 + 1)(x^4 + 1)$ with respect to x.

29. Differentiate $\frac{x^8-1}{x^4-1}$ with respect to x.

Differentiate the following with respect to x using First principle method. (For Q. 30 – 35)

30. $\frac{1}{x}$

31. $\sqrt{x}$

32. $\cos(x+1)$

33. $\sqrt{\sin x}$

34. $\frac{2x+3}{x+1}$

35. $x \cos x$

Long Answer Type Questions

Evaluate the following Limits: (For Q. 36 – 58)

36. $\lim_{x\to\infty} \frac{2x^8 - 3x^2 + 1}{x^8 + 6x^5 - 7}$

37. $\lim_{x\to 2} \frac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$

38. $\lim_{x\to 0} \frac{1 - \cos 2x}{x \cdot \tan 3x}$

39. $\lim_{x\to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

40. $\lim_{x\to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{\frac{\pi}{6} - x}$

41. $\lim_{x\to 0} \frac{\sin x}{\tan x^0}$ (where x^0 represents x degree)

42. $\lim_{x\to 9} \frac{x^{\frac{3}{2}} - 27}{x^2 - 81}$

43. $\lim_{x\to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x - a}$

44. $\lim_{x\to 0} \frac{\cos ax - \cos bx}{1-\cos x}$

45. $\lim_{x\to a} \frac{\cos x - \cos a}{\cot x - \cot a}$

46. $\lim_{x\to \pi} \frac{1+\sec^3 x}{\tan^2 x}$

47. $\lim_{x\to 1} \frac{x-1}{\log_e x}$

48. $\lim_{x\to e} \frac{x-e}{(\log_e x)-1}$

49. $\lim_{x\to 2} \left[\frac{4}{x^3-2x^2} + \frac{1}{2-x}\right]$

50. $\lim_{x\to a} \left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

51. $\lim_{x\to 0} \frac{\sin(2+x)-\sin(2-x)}{x}$

52. $\lim_{x\to 0} \frac{1-\cos x \cdot \sqrt{\cos 2x}}{\sin^2 x}$

53. $\lim_{x\to 0} \frac{6^x - 2^x - 3^x + 1}{\log(1+x^2)}$

54. $\lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x}$

55. $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{1-\sqrt{2}\sin x}$

56. Find the values of a and b if $\lim_{x\to 2} f(x)$ and $\lim_{x\to 4} f(x)$ exists where

$$f(x) = \begin{vmatrix} x^2 + ax + b, & 0 \le x < 2 \\ 3x + 2, & 2 \le x \le 4 \\ 2ax + 5b, & 4 < x < 8 \end{vmatrix}$$

57. Differentiate the following w.r.t.

(a) $\dfrac{(x-1)(x-2)(x-3)}{x^2 - 5x + 6}$

(b) $\left(x - \dfrac{1}{x}\right)\left(x + \dfrac{1}{x}\right)\left(x^2 + \dfrac{1}{x^2}\right)\left(x^4 + \dfrac{1}{x^4}\right)$

(c) $\dfrac{x \sin x + \cos x}{x \sin x - \cos x}$

(d) $x \cdot \sin x \cdot e^x$

58. Prove the following statements

(a) If $y = \dfrac{x}{x+2}$, then $\dfrac{dy}{dx} = \dfrac{(1-y)y}{x}$

(b) If $y = e^x \cos x$, then $\dfrac{dy}{dx} = \sqrt{2}\, e^x \cos\left(x + \dfrac{\pi}{4}\right)$

(c) If $y = \dfrac{1-x}{1+x}$, then $\dfrac{dy}{dx} = \dfrac{-2}{(1+x)^2}$

(d) If $xy = 4$, then $x\left(\dfrac{dy}{dx} + y^2\right) = 3y$

CASE STUDY TYPE QUESTIONS

59. Mr. Pradeep has a rectangular plot, which is used for growing vegetables. Perimeter of plot is 50m. Length and width of plot are x m and y m respectively.

Based on the above information, answer the following questions:-

i. Relation between x and y is

(a) $x + y = 50$ (b) $x + y = 100$

(c) $x + y = 25$ (d) $x = y$

ii. Area function, A(x) =

(a) $x^2 - 5$ (b) $25x - x^2$ (c) $x^2 - 25x$ (d) $25 - x$

iii. Derivative of A(x) w.r.t. x [A'(x)] =

(a) $2x$ (b) $-2x$ (c) $25 - 2x$ (d) $2x - 25$

iv. Value of x for which A'(x) = 0 is

(a)25 (b) 12.5 (c) 5 (d) 0

v. Value of A(x) at x = 12.5 is

(a) 625 (b) 250 (c) 156.25 (d) 144.25

60. Consider the following functions.

$u(x) = \sqrt{x}, \quad v(x) = \cot x, \quad f(x) = u(x) \times v(x)$

$g(x) = \dfrac{u(x)}{v(x)}$ and $h(x) = \dfrac{v(x)}{u(x)}$

Based on the above information answer the following :-

i. Derivative of $u(x)$ is

(a) $\dfrac{1}{\sqrt{x}}$ (b) $\dfrac{2}{\sqrt{x}}$ (c) $\dfrac{\sqrt{x}}{2}$ (d) $\dfrac{1}{2\sqrt{x}}$

ii. Derivative of $v(x)$ is

(a) $-\text{cosec}\, x \cot x$ (b) $-\text{cosec}^2 x$ (c) $\sec^2 x$ (d) $\tan x$

iii. Derivative of $f(x)$ is

(a) $\dfrac{-2x\text{cosec}^2 x + \cot x}{2\sqrt{x}}$ (b) $\dfrac{-\text{cosec}^2 x}{2\sqrt{x}}$

(c) $\dfrac{-2x\cot^2 x + \text{cosec} x}{2\sqrt{x}}$ (d) $\dfrac{-\cot^2 x}{2\sqrt{x}}$

iv. Derivative of $g(x)$ is

(a) $\dfrac{\cot x - 2x\text{cosec}^2 x}{2\sqrt{x}\cot^2 x}$ (b) $\dfrac{\cot x + 2x\text{cosec}^2 x}{2\sqrt{x}\cot^2 x}$

(c) $\dfrac{\text{cosec}^2 x - 2x\cot^2 x}{2\sqrt{x}\cot^2 x}$ (d) $\dfrac{\text{cosec}^2 x + 2x\cot^2 x}{2\sqrt{x}\cot^2 x}$

v. Derivative of $h(x)$ is

(a) $\dfrac{2x\text{cosec}^2 x + \cot x}{2x^{3/2}}$ (b) $\dfrac{\text{cosec}^2 x + 2x\cot x}{2x^{3/2}}$

(c) $\dfrac{-(2x\text{cosec}^2 x + \cot x)}{2x^{3/2}}$ (d) $\dfrac{-\text{cosec}^2 x}{\sqrt{x}}$

Multiple Choice Questions

Note: Q.61 – Q.70 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

61. $\lim\limits_{x\to\pi} \dfrac{\sin x}{x-\pi}$ is -

(a) 1 (b) 2
(c) –1 (d) does not exist.

62. If $\lim_{x\to 2}\frac{x^n-2^n}{x-2}=80,$ then n is -
(a) 2 (b) 3
(c) 4 (d) 5.

63. If $L=\lim_{x\to 1}\frac{x^4-1}{x^3-1},$ then 3L is -
(a) 2 (b) 3
(c) 4 (d) None of these.

64. $\lim_{x\to 0}\frac{(1+x)^{16}-1}{(1+x)^4-1}$ is -
(a) 0 (b) 4
(c) 8 (d) 16.

65. $\lim_{x\to 1}\frac{x+x^2+x^3+x^4-4}{x-1}$ is -
(a) 0 (b) 4
(c) 10 (d) Does not exist.

66. $\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1}$ is -
(a) 0 (b) 1
(c) 2 (d) 4.

67. $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$ is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $\frac{2}{3}$ (d) 1

68. $\lim_{x\to 0}(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 1

69. If $y=\sin^4 x+\cos^4 x$, then $\frac{dy}{dx}=$

(a) $4\sin^3x + 4\cos^3x$ (b) $4\sin^3x - 4\cos^3x$

(c) $-\sin 4x$ (d) 0.

70. If $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$ then $\frac{dy}{dx}$ is

(a) y^2 (b) $1+y^2$

(c) y^2-1 (d) $1-y^2$

ANSWERS

1. 15
2. 4
3. 32
4. 8
5. 9
6. $\frac{1}{8}$
7. $\frac{1}{2}$

8. $\log 2$

9. $\log_3 5$

10. –2 log 3

11. –3

12. log 7

13. 2x – sinx

14. 2x – 1

15. $\dfrac{4x^5 - 15x^4}{(x-3)^2}$

16. 4

17. 1

18. $\dfrac{1}{4}$

19. $\cos^2 x . \sin x(2\cos^2 x - 3\sin^2 x)$

20. 2cos2x.cos3x – 3sin2x.sin3x

21. $\dfrac{x\cos x - \sin x}{x^2}$

22. $3x^2 + 3x.\log 3$

23. 0

24. 0

25. 0

26. 1

27. $\dfrac{8}{3}$

28. $8x^7$

29. $4x^3$

30. $\dfrac{-1}{x^2}$

31. $\dfrac{1}{2\sqrt{x}}$

32. –sin(x + 1)

33. $\frac{1}{2}\cot x\sqrt{\sin x}$

34. $\frac{-1}{(x+1)^2}$

35. $\cos x - x\sin x$

36. 2

37. $\frac{1}{2}$

38. $\frac{2}{3}$

39. $\sqrt{2}$

40. 2

41. $\frac{180^0}{\pi}$

42. $\frac{1}{4}$

43. $\frac{5(a+2)^{\frac{3}{2}}}{2}$

44. $b^2 - a^2$

45. $\sin^3 a$

46. $\frac{-3}{2}$

47. 1

48. e

49. –1

50. $\frac{1}{\sqrt{3}}$

51. 2cos2

52. $\frac{3}{2}$

53. (log2)(log3)

54. $\frac{1}{2}$

55. 2

56. a = –1, b = 6

57. (a) 1 (b) $8x^7 + 8x^{-9}$

(c) $\frac{-2(x + \sin x \cdot \cos x)}{(x\sin x - \cos x)^2}$ (d) e^x (xsinx + xcosx + sinx)

59. i. (c) ii. (b) iii. (c) iv. (b) v. (c)

60. i. (d) ii. (b) iii. (a) iv. (b) v. (c)

61. (c)

62. (d)

63. (c)

64. (b)

65. (c)

66. (c)

67. (b)

68. (a)

69. (c)

70. (d)

CHAPTER - 15

STATISTICS

KEY CONCEPT

- Range of Ungrouped Data and Discrete Frequency Distribution.
- RANGE = Largest observation – smallest observation.
- Range of Continuous Frequency Distribution.
- RANGE= Upper Limit of Highest Class – Lower Limit of Lowest Class.
- **Mean deviation for ungrouped data or raw data:**

$$M.D. \text{ (about mean)} = \frac{\sum |x_i - \bar{x}|}{n}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D. \text{ (about median)} = \frac{\sum |x_i - M|}{n}, \text{ where } M \text{ is the Median.}$$

- **Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution):**

$$M.D. \text{ (about mean)} = \frac{\sum f_i |x_i - \bar{x}|}{N}, \text{ where } \bar{x} \text{ is the Mean.}$$

$$M.D. \text{ (about mean)} = \frac{\sum f_i |x_i - M|}{N}, \text{ where } M \text{ is the Median.}$$

Note: $N = \sum f_i$

- Variance is defined as the mean of the squares of the deviations from mean.

- Standard deviation 'σ' is positive square root of variance.
  $\sigma = \sqrt{\text{Variance}}$

- Variance 'σ^2' and standard deviation (SD) σ for ungrouped data
  $$\sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 \Rightarrow \boxed{S.D. = \sigma = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}}$$

- Standard deviation of a discrete frequency distribution
  $$S.D. = \sigma = \sqrt{\frac{1}{N}\sum f_i (x_i - \bar{x})^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - (\sum f_i x_i)^2}$$

- Short cut method to find variance and standard deviation
  $$\text{Variance} = \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$
  $$S.D. = \sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$
  $$\text{Where : } y_i = \frac{x_i - A}{h}, \text{ A = Assumed mean}$$

- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k^2 times of the original value and standard deviation becomes k times of the original value.

- If each observation is increased by k, where k is positive or negative, then variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.

- The mean of first 'n; natural number is $\frac{n+1}{2}$.
- The mean of first 'n' even natural numbers = (n + 1)

VERY SHORT ANSWER TYPE QUESTIONS

1. The sum of the squares of deviation for 10 observations taken from their mean 50 is 250. Find Standard Deviation.
2. The sum of the squares of deviation for 10 observations taken from their mean 25 is 500. Find Variance.
3. If the variance of 14, 18, 22, 26, 30 is 'k', then find the variance of 28, 36, 44, 52, 60.

SHORT ANSWER TYPE QUESTIONS

4. Find the Variance of First 10 Natural Numbers.
5. Find the Variance of First 5 Multiples of 6.
6. Find the Standard Deviations of First 10 Even Natural numbers.
7. Find the Standard deviation for the following data:

   10, 20, 30, 40, 50, 50, 60, 70, 80, 90

8. Find the variance for the following Data:

Class-Interval	Frequency
0 - 10	1
10 - 20	2
20 – 30	3
30 – 40	3

40 – 50	1

LONG ANSWER TYPE – I QUESTIONS

9. In a series of '2p' observations, half of the observations are equal 'a' each and remaining half equal (–a) each. If the standard deviation of the observations is 2, then find the value of |a|.

10. In the following Distribution

x	f
A	2
2A	1
3A	1
4A	1
5A	1
6A	1

Where A is positive integer, has a variance of 160. Determine the value of A.

11. Find the mean deviation from mean of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

12. Find the Variance and Standard Deviation of first n terms of an Arithmetic Progression (A.P.) with first term is 'a' and Common difference is 'd'.

13. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, find the variance of the numbers so obtained.

14. Two sets each of 20 obvservations, have the same standard deviation 5. The first set has a mean 17 and the second a mean

22. Determine the SD of the set obtained by combining the given two sets.

15. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observationsare 1, 2 and 6. Find the other two observations.

16. Calculate the possible values of 'x' if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

17. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

18. Suppose a population A has 100 observations 101, 102,......, 200. Another population B has 100 observations 151, 152,......., 250. If $V_A$and $V_B$represent the variances of the two populations respectively then find the ratio of $V_A$and V_B.

LONG ANSWER TYPE – II QUESTIONS

19. Calculate the mean deviation about mean for the following data.

X	2	4	6	8	10	12	14	16
f	2	2	4	5	3	2	1	1

20. If for a distribution $\sum(x-5)=3,\ \sum(x-5)^2=43$ and the total number of item is 18, find the mean and standard deviation.

21. Calculate the mean deviation about median for the following data:

X	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	4

22. There are 60 students in a class. The following is the frequency distribution of the marksobtained by the students in a test :

X	0	1	2	3	4	5
f	p – 2	p	p^2	$(p + 1)^2$	2p	2p + 1

where p is positive integer. Determine the mean and standard deviation of the marks.

23. Calculate the mean deviation about mean

Class Interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
f	2	3	8	14	8	3	2

24. Mean and standard deviation of 100 observations were found to be 40 and 10respectively. If at the time of calculation two observations were wrongly taken as 30 and70 in place of 3 and 27 respectively. Find correct standard deviation.

25. Calculate the mean deviation about mean for the following data:

Class Interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
f	5	8	15	16	6

26. Calculate the mean deviation about median for the following data:

Class Interval	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80	80 - 90
f	8	10	10	16	14	2

27. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results:

Number of observations = 25,
mean = 18.2 seconds
Standard deviation = 3.25 seconds

Further another set of 15 observations x_1, x_2, ……….…., x_{15}, also in $\sum_{i=1}^{15} x_i^2 = 5524$.

Calculate the standard deviation based on all 40 observations.

28. Find the mean deviation about mean of the following data:

Class Interval	f
20 - 29	5
30 - 39	12
40 - 49	15
50 - 59	20
60 - 69	18
70 - 79	10
80 - 89	6
90 - 99	4

CASE STUDY TYPE QUESTIONS

29. Following data represents the salaries of 11 employees in a firm

10000, 12000, 15000, 13000, 11000, 12000, 12000, 14000, 10000, 13000, 12000.

i. Find the mean salary.

(a) 11181.82 (b) 12181.82

(c) 13181.82 (d) 10000.82

ii. What is the median salary?

(a) 12000 (b) 11000

(c) 12181.82 (d) 11181.82

iii. When arranged in ascending order, which entry gives the median salary?

(a)6^{th} (b) 5^{th} (c) 4^{th}(d) 7^{th}

iv. The mean deviation about the median salary is

(a)1190.99 (b) 1000 (c) 1100 (d) 1090.91

v. What is the range of salaries?

(a)4500 (b) 4000 (c) 5000 (d) 6000

30. Following are the prices of shares X and Y (of ten days) :

Days	**X**	**Y**
1	35	108
2	54	107
3	52	105
4	53	105
5	56	106
6	58	107
7	52	104
8	50	103
9	51	104
10	49	101

i. What is the mean prince of the share X during these 10 days?

(a)52 (b) 51 (c) 50.3 (d) 51.5

ii. What is the mean prince of the share Y during these 10 days?

(a)105 (b) 106 (c) 104 (d) 107

iii. What is the standard deviation of the price of share X?

(a)5.01 (b) 6.75 (c) 5.92 (d) 7.25

iv. What is the standard deviation of the price of share Y?

(a) 1.75 (b) 2.87 (c) 1.25 (d) 2

v. If a person wants to invest in shares (X or Y) whose price remain more stable. He should invest in

(a) X

(b) Y

(c) Both are equally stable. So, he can invest iin anyone

(d) Insufficient data to decide

Multiple Choice Questions

Note: Q.31–Q.40 are Multiple Choice Questions (MCQ), select the correct alternatives out of given four alternatives in each.

31. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new mean -

    (a) 12 (b) 16

    (c) 24 (d) 48.

32. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what is the new standard deviation -

    (a) 4 (b) 8

    (c) 16 (d) 32.

33. The standard deviation of 25 observations is 4 and their mean is 25. If each observation isincreased by 10, what is the new mean-

    (a) 25 (b) 29

    (c) 30 (d) 35.

34. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what is the new variance -

    (a) 4 (b) 14

    (c) 16 (d) 25.

35. Match the following:

If the mean of $x_1, x_2, \ldots\ldots, x_{20}$ is 10.

	Column-1		Column-2
A	mean of $2x_1, 2x_2, \ldots\ldots, 2x_{20}$	P	0
B	mean of $(-3x_1 + 32), (-3x_2 + 32), \ldots\ldots, (3x_{20} + 32)$	Q	2
C	mean of $(x_1 + 2), (x_2 + 2), \ldots\ldots, (x_{20} + 2)$	R	12
D	mean of $(x_1 - 10), (x_2 - 10), \ldots\ldots, (x_{20} - 10)$	S	20

(a) A → P, B → Q, C → R, D → S

(b) A → S, B → Q, C → R, D → P

(c) A → Q, B → S, C → R, D → P

(d) A → S, B → Q, C → P, D → R

36. If mean of first n natural numbers is $\frac{5n}{9}$, then n =

(a) 5 (b) 4

(c) 9 (d) 10

37. Find the mean of 6, 7, 10, 12, 13, 4, 8, 12

(a) 9 (b) 10

(c) 12 (d) 13

38. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is

(a) 2.23 (b) 2.57

(c) 3.23 (d) 3.57

39. The following information relates to a sample of size 60 : $\Sigma x^2 = 18000, \Sigma x = 960$

The variance is

(a) 6.63 (b) 16

(b) 22 (d) 44

40. The standard deviation of the data 6, 5, 9, 13, 12, 8, 10 is

(a) $\sqrt{\frac{52}{7}}$ (b) $\frac{52}{7}$

(c) $\sqrt{6}$ (d) 6

ANSWERS

1. 5
2. 50
3. 4 k
4. 8.33
5. 72
6. $\sqrt{33}$
7. $10\sqrt{6}$
8. $\sqrt{129}$
9. 2

10 A = 7

11. $\frac{(n-1)(d-1)}{2}$

12. Variance = $\frac{(n^2-1)}{12}d^2$

    Standard Deviation = $d\sqrt{\frac{(n^2-1)}{12}}$

13. 8.25
14. 5.59
15. 4, 9
16. 3, $\frac{7}{3}$
17. 6.5, 2.5
18. 1 : 1

19. 2.8

20. Mean = 5.17,
Standard Deviation = 1.53

21. 10.1

22. Mean = 2.8,
Standard deviation = 1.12

23. 10

24. 10.24

25. 9.44

26. 11.44

27. 3.87

29. i. (b) ii. (a) iii. (a) iv. (d) v. (c)

30. i. (b) ii. (a) iii. (c) iv. (d) v. (b)

31. (d)

32. (c)

33. (d)

34. (c)

35. (b)

36. (c)

37. (a)

38. (b)

39. (d)

40. (a)

CHAPTER - 16

PROBABILITY

KEY CONCEPT

- **Random Experiment:** If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.
- **Sample Space:** The collection or set of all possible outcomes of a random experiment is called sample space associated with it.Each element of the sample space (set) is called a sample point.
- **Event:** A subset of the sample space associated with a random experiment is called an event.
- **Elementary or Simple Event:** An event which has only one Sample point is called a simple event.
- **Compound Event:** An event which has more than one Sample point is called a Compound event.
- **Sure Event:** If an event is same as the sample space of the experiment, then event is called sure event. In other words an event which is certain to happen is sure event.
- **Impossible Event:** Let S be the sample space of the experiment, $\phi \subset S$, ϕ is called impossible event. In other words an event which is impossible to be happen is the impossible event.
- Exhaustive and Mutually Exclusive Events: If Events E_1, E_2, E_3……..E_n are n events of a sample space S such that
  (i) $E_1 U\ E_2 U E_3 U$…….. $UE_n = S$ then Events E_1, E_2, E_3……..E_n are called exhaustive events.

(ii) $E_i \cap E_j = \phi$ for every $i \neq j$ then Events $E_1, E_2, E_3 \ldots\ldots E_n$ are called mutually exclusive.

- **Probability of an Event:** For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

$$P(A) = \frac{n(A)}{n(S)}$$

where n(A) is number of elements in A and n(S) is number of elements in set S and $0 \le P(A) \le 1$

(a) If A and B are any two events then

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A \text{ and } B)$

(b) A and B are mutually exclusive events, then

$P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B) = 0$ for mutually exclusive events)

(c) $P(A) + P(\bar{A}) = 1$ or P(A) + P(not A) = 1

(d) P (Sure event) = P(S) = 1

(e) P (impossible event) = $P(\phi) = 0$

(f) $P(A - B) = P(A) - P(A \cap B) = P(A \cap \bar{B})$

(g) $P(B - A) = P(B) - P(A \cap B) = P(\bar{A} \cap B)$

(h) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

(i) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

- **Addition theorem for three events**

  Let A, B and C be any three events associated with a random experiment, then

  $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- **Axiomatic Approach to Probability:**

  Let S be a sample space containing elementary outcomes w_1, w_2, ..., w_n

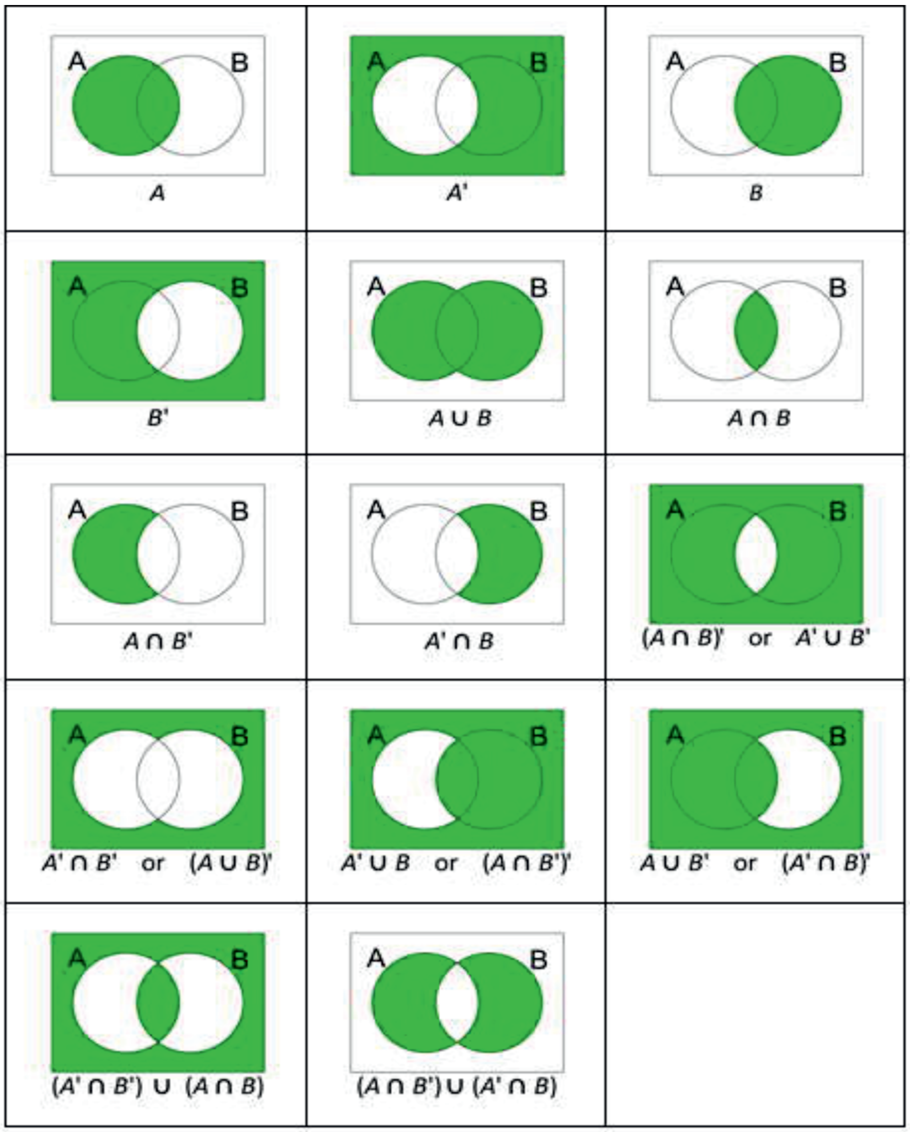
  i.e. S = {w_1, w_2, ..., w_n}

  (i) $0 \le P(w_i) \le 1, \text{ for all } w_i \,\varepsilon\, S$

  (ii) $P(w_1) + P(w_2) + P(w_3) + \ldots\ldots + P(w_n) = 1$

  (iii) P (A) = $\sum P(w_i)$, for any event A containing elementary events w_i.

▶ VENN DIAGRAM OF DIFFERENT SETS



- The cards J, Q and K are called face cards. There are 12 face cards in a deck of 52 cards.
- There are 64 squares in a chess board i.e.32 white and 32 Black.

VERY SHORT ANSWER TYPE QUESTIONS

1. Describe the Sample Space for the experiment:

   A coin is tossed twice and number of heads is recorded.

2. Describe the Sample Space for the experiment:
   A card is drawn from a deck of playing cards and its colour is noted.

3. Describe the Sample Space for the experiment:
   A coin is tossed repeatedly until a tail comes up.

4. Describe the Sample Space for the experiment:
   A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.

5. Describe the Sample Space for the experiment:
   Two balls are drawn at random in succession without replacement from a box containing 1 red and 3 identical white balls.

6. A coin is tossed n times. Find the number of element in its sample space.

7. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?

8. What is the probability that a given two-digit number is divisible by 15?

9. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?

10. If $P(A \cup B) = P(A \cap B)$, then find relation between P(A) and P(B).

SHORT ANSWER TYPE QUESTIONS

11. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$, find P(B) if $P(A \cap B) = P(A)\,P(B)$.

12. Three identical dice are rolled. Find the probability that the same number appears on each of them.

13. In an experiment of rolling of a fair die. Let A, B and C be three events defined as under:

A : a number which is a perfect square

B : a prime number

C : a number which is greater than 5.

Is A, B, and C exhaustive events?

14. Punching time of an employee is given below:

DAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
TIME (AM)	10:35	10:20	10:22	10:27	10:25	10:40

If the reporting time is 10:30 a.m, then find the probability of his coming late.

15. A game has 18 triangular blocks out of which 8 are blue and rest are red and 19 square blocks out of which 7 are blue and rest are yellow. On piece is lost. Find the probability that it was a square of blue colour.

16. A card is drawn from a pack of 52 cards. Find the probability of getting:
    (i) a jack or a queen
    (ii) a king or a diamond
    (iii) a heart or a club
    (iv) either a red or a face card.
    (v) neither a heart nor a king
    (vi) neither an ace nor a jack
    (vii) a face card

17. In a leap year find the probability of
    (i) 53 Mondays and 53 Tuesdays
    (ii) 53 Mondays and 53 Wednesday

(iii) 53 Mondays or 53 Tuesdays

(iv) 53 Mondays or 53 Wednesday

18. In a non-leap year, find the probability of

    (i) 53 Mondays and 53 Tuesdays.

    (ii) 53 Mondays or 53 Tuesdays.

19. Three candidates A, B, and C are going to play in a chess competition to win FIDE (World Chess Federation) cup this year. A is thrice as likely to win as B and B is twice as likely as to win C. Find the respective probability of A, B and C to win the cup.

LONG ANSWER QUESTIONS

20. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I′s come together.

21. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn atrandom. If the probability of both of them being blue is $\frac{5}{14}$, find x.

22. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

23. Find the probability of at most two tails or at least two heads in a toss of three coins.

24. A, B and C are events associated with a random experiment such that

    P(A) = 0.3,

    P(B) = 0.4, P(C) = 0.8, P(A ∩ B) = 0.08, P(A ∩ C) =0.28 and

    P(A∩B∩C) = 0.09. If

    P(A U B U C) ≥ 0.75 Then prove that P(B ∩ C) lies in the interval

    [0.23, 0.48].

25. $\frac{1+3p}{3}$, $\frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probability of three mutually exclusive events. Thenfind the set of all values of p.

26. An urn A contains 6 red and 4 black balls and urn B contains 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at randomfrom urn B and placed in urn A. Now if one ball is drawn at random from urn A then findthe probability that it is found to be red.

27. If three distinct numbers are chosen randomly from the first 100 natural numbers, then find the probability that all three of them are divisible by both 2 and 3.

28. S = {1, 2, 3, ………, 30}, A = {x : x is multiple of 7}, B = { x : x is multiple of 5},C = {x : x is a multiple of 3}.

    If x is a member of S chosen at random find the probability that

    (i) $x \in A \cup B$

    (ii) $x \in B \cap C$

    (iii) $x \in A \cap \bar{C}$

29. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.

30. If A and B are any two events having $P(A \cup B) = \frac{1}{2}$ and $P(\bar{A}) = \frac{2}{3}$, then find the $P(\bar{A} \cap B)$.

31. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?

32. A typical PIN (Personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

33. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.

34. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that

(i) it will get at least one of the two awards

(ii) it will get only one of the awards.

35. A girl calculates that the probability of her winning the first prize in a lottery is 0.02. If 6000 tickets were sold, how many tickets has she bought?

36. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9?

37. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.

38. If A,B and C are three mutually exclusive and exhaustive events of an experiment such that
$3P(A) = 2P(B) = P(C)$, then find the value of $P(A)$.

CASE STUDY TYPE QUESTIONS

39. To make a healthy routine and to do some physical exercise during lockdown a family decided to roll a dice and based on the outcomes, they will decide activities to be done.

- If the outcome is 2, 4 or 6, they will do 30 minutes walk on the roof.
- If it shows 1 or 3 on the dice, 15 minutes meditation to be done.
- If outcome is 5, then they will toss a coin. If it shows "Head", the family will do 5 minutes of rope skipping. If there is "Tail", family will do 20 minutes of Yoga.

i. How many elements are there in the sample space?

(a)6 (b) 9 (c) 7 (d) 8

ii. What is the probability of doing walking?

(a) $\frac{5}{6}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{6}$

iii. What is the probability of doing rope skipping?

(a) $\frac{1}{3}$ (b) $\frac{2}{7}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

iv. What is the probability of doing yoga or meditation?

(a) 1 (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{1}{2}$

v. Two activities having the same probability are

(a) Walking and Yoga

(b) Yoga and Rope Skipping

(c) Rope Skipping and Walking

(d) Walking and Meditation

40. In a class of 60 students, hobbies were discussed. 30 liked reaing, 32 liked singing and 24 liked about reading and singing.

i. Find the probability that the student liked reading or singing.

(a) $\frac{17}{30}$ (b) $\frac{19}{30}$ (c) $\frac{23}{30}$ (d) $\frac{29}{30}$

ii. How many students neither like reading nor singing?

(a) 30 (b) 28 (c) 22 (d) 38

iii. Find the probability that the student neither like singing nor reading?

(a) $\frac{11}{30}$ (b) $\frac{13}{30}$

(c) $\frac{7}{30}$ (d) $\frac{1}{30}$

iv. Find the probability that a student like singing but not reading?

(a) $\frac{4}{15}$ (b) $\frac{7}{15}$ (c) $\frac{1}{15}$ (d) $\frac{2}{15}$

v. Find the probability that a student like reading only.

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{7}{10}$ (d) 0

Multiple Choice Questions

41. Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is -

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$

(c) $\frac{5}{9}$ (d) $\frac{1}{30}$.

42. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

(a) $\frac{1}{16}$ (b) $\frac{16}{25}$

(c) $\frac{1}{65}$ (d) $\frac{1}{25}$.

43. The probability that a non-leap year selected at random will have 52 Sundays is -

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

44. The probability that a non-leap year selected at random will have 53 Sundays is -

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

45. The probability that a leap year selected at random will have 54 Sundays is

(a) 0 (b) 1

(c) $\frac{1}{7}$ (d) $\frac{2}{7}$.

46. Three unbiased coins are tossed. If the probability of getting at least 2 tails is p, Thenthe value of 8p -

(a) 0 (b) 1

(c) 3 (d) 4.

47. Four unbiased coins are tossed. If the probability of getting odd number of tails is p, then the value of 16p -

(a) 1 (b) 2

(c) 4 (d) 8

48. From 4 red balls, 2 white balls and 4 black balls, four balls are selected. The probability of getting 2 red balls is p, then the value of 7p -

(a) 1 (b) 2

(c) 3 (d) 4

49. If A and B are mutually exclusive events, then

(a) $P(A) \leq P(\bar{B})$ (b) $P(A) \geq P(\bar{B})$

(c) $P(A) < P(\bar{B})$ (d) None of these

50. The probability that atleast one of the events A and B occur simultaneously with probability 0.2, then $P(\bar{A}) + P(\bar{B})$ is

(a) 0.4 (b) 0.8

(c) 1.2 (d) 1.6

ANSWERS

1. {0, 1, 2}
2. {R, B}
3. {T, HT, HHT, ...}
4. {HR_1, HR_2, HB_1, HB_2, HB_3, TH, TT}
5. {RW, WW, WR}
6. 2^n
7. $\frac{8}{21}$
8. $\frac{1}{15}$
9. Mutually Exclusive
10. P(A) = P(B)
11. $\frac{5}{7}$
12. $\frac{1}{36}$
13. Yes, A, B and C are Exhaustive Events

14. $\frac{1}{3}$

15. $\frac{7}{37}$

16. (i) $\frac{2}{13}$ (ii) $\frac{4}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{8}{13}$

(v) $\frac{9}{13}$ (vi) $\frac{11}{13}$ (vii) $\frac{3}{13}$

17. (i) $\frac{1}{7}$ (ii) 0 (iii) $\frac{3}{7}$ (iv) $\frac{4}{7}$

18. (i) 0 (ii) $\frac{2}{7}$

19. $P(A) = \frac{6}{9} = \frac{2}{3}, P(B) = \frac{2}{9}, P(C) = \frac{1}{9}$

20. $\frac{1}{5}$

21. 3

22. $\frac{23}{28}$

23. $\frac{7}{8}$

24. $0.23 \le P(B) \le 0.48$

25. $\frac{-1}{3} \le P \le \frac{-1}{3}$

26. $\frac{32}{55}$

27. $\frac{4}{1155}$

28. (i) $\frac{1}{3}$ (ii) $\frac{1}{15}$ (iii) $\frac{1}{10}$

29. $\frac{3}{10}$

30. $\frac{1}{6}$

31. $\frac{1}{10}$

32. $\frac{265896}{1679616}$

33. $\frac{63}{190}$

34. (i) 0.64
    (ii) 0.44

35. 120

36. $\frac{5}{12}$

37. $\frac{3}{5}$

38. $\frac{2}{11}$

39. i. (c) ii. (b) iii. (d) iv. (c) v. (d)

40. i. (b) ii. (c) iii. (a) iv. (d) v. (a)

41. (c)

42. (d)

43. (b)

44. (c)

45. (a)

46. (d)

47. (d)

48. (c)

49. (a)

50. (c)

Sample Question Paper- 1
Class XI
Session 2022-23
Mathematics (Code-041)

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A
(Multiple Choice Questions)

Each question carries 1 mark

1. Which of the following is the empty set

(a) $\{x : x$ is a real number and $x^2 - 1 = 0\}$

(b) $\{x : x$ is a real number and $x^2 + 1 = 0\}$

(c) $\{x : x$ is a real number and $x^2 - 9 = 0\}$

(d) $\{x : x$ is a real number and $x^2 = x + 2\}$

2. The number of proper subsets of the set $\{1, 2, 3\}$ is

(a) 8 (b) 7

(c) 6 (d) 5

3. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to

(a) A (b) B

(c) ϕ (d) None of these

4. Let $A = \{1, 2, 3\}$. The total number of distinct relations that can be defined over A is

(a) 2^9 (b) 6

(c) 8 (d) None of these

5. Let R be a relation on N defined by $x + 2y = 8$. The domain of R is

(a) {2, 4, 8} (b) {2, 4, 6, 8}

(c) {2, 4, 6} (d) {1, 2, 3, 4}

6. If $f(x) = \dfrac{x - |x|}{|x|}$, then $f(-1) =$

(a) 1 (b) –2

(c) 0 (d) +2

7. If $\tan\theta = \dfrac{-4}{3}$, then $\sin\theta =$

(a) – 4/5 but not 4/5 (b) – 4/5 or 4/5

(c) 4/5 but not – 4/5 (d) None of these

8. The complex number $\dfrac{1+2i}{1-i}$ lies in which quadrant of the complex plane

(a) First (b) Second

(c) Third (d) Fourth

9. In how many ways can 5 boys and 3 girls sit in a row so that no two girls are together

(a) $5! \times 3!$ (b) ${}^4P_3 \times 5!$

(c) ${}^6P_3 \times 5!$ (d) ${}^5P_3 \times 3!$

10. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, then the value of r is

(a) 3 (b) 4

(c) 5 (d) 8

11. If coefficient of $(2r+3)^{th}$ and $(r-1)^{th}$ terms in the expansion of $(1+x)^{15}$ are equal, then value of r is

(a) 5 (b) 6

(c) 4 (d) 3

12. The equation of the straight line passing through the point (3, 2) and perpendicular to the line $y = x$ is

(a) $x - y = 5$ (b) $x + y = 5$

(c) $x + y = 1$ (d) $x - y = 1$

13. The distance between the foci of the ellipse $3x^2 + 4y^2 = 48$ is

(a) 2 (b) 4

(c) 6 (d) 8

14. If $\lim\limits_{x \to 2} \dfrac{x^n - 2^n}{x - 2} = 80$, where n is a positive integer, then $n =$

(a) 3 (b) 5

(c) 2 (d) None of these

15. $\lim_{x \to 0} \frac{1-\cos mx}{1-\cos nx} =$

(a) m/n (b) n/m

(c) $\frac{m^2}{n^2}$ (d) $\frac{n^2}{m^2}$

16. Let the function f be defined by the equation $f(x) = \begin{cases} 3x & \text{if } 0 \le x \le 1 \\ 5-3x & \text{if } 1 < x \le 2 \end{cases}$, then

(a) $\lim_{x \to 1} f(x) = f(1)$ (b) $\lim_{x \to 1} f(x) = 3$

(c) $\lim_{x \to 1} f(x) = 2$ (d) $\lim_{x \to 1} f(x)$ does not exist

17. There are two children in a family. The probability that both of them are boys is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) None of these

18. Two dice are thrown simultaneously. The probability of getting the sum 2 or 8 or 12 is

(a) $\frac{5}{18}$ (b) $\frac{7}{36}$

(c) $\frac{7}{18}$ (d) $\frac{5}{36}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. **Assertion :** If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24th position.

Reason : The number of ways of arranging four distinct objects taken all at a time is C(4, 4).

20. **Assertion :** A letter is chosen at random from the word NAGATATION. Then, the total number of outcomes is 10.
Reason : A letter is chosen at random from the word 'ASSASSINATION' Then, the total number of outcomes is 13.

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Prove that : $\sin 780° \sin 120° + \cos 240° \sin 390° = \frac{1}{2}$

22. Find the values of $\tan \frac{13\pi}{12}$

OR

Prove that : $\frac{\cos 11^o + \sin 11^o}{\cos 11^o - \sin 11^o} = \tan 56^o$

23. Solve $x + 5 > 2(x + 1), 2 - x < 3(x + 2)$

24. A coin is tossed repeatedly until a head comes for the first time. Describe the sample space.

25. Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?

OR

If A and B are two events associated with a random experiment such that $P(A \cup B) = 0.8$, $P(A \cap B) = 0.3$ and $P(\overline{A}) = 0.5$, find P(B).

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. A market research group conducted a survey of 2000 consumers and reported that 1720 consumers liked product P_1 and 1450 consumers liked product P_2. What is the least number that must have liked both the products ?

27. If $(x+iy)^{1/3} = a + ib$, x , y, a b $\in$ R. Show that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$.

OR

Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

28. A solution is to be kept between 86° and 95°F. What is the range of temperature in degree Celsius, if the Celsius (C)/Fahrenheit (F) conversion formula is given by $F=\frac{9}{5}C+32$.

29. Find the coefficient of x^5 in the expansion of the product $(1 + 2x)^6 (1 - x)^7$.

OR

Find the term independent of x in the expansion of $\left(3x^2-\frac{1}{2x^3}\right)^{10}$

30. Find the equation of a circle of radius 5 whose centre lies on x–axis and passes through the point (2, 3).

OR

Find the equation of an ellipse whose axes lie along coordinate axes and which passes through (4,3) and (-1, 4).

31. Find the locus of the point, the sum of whose distances from the points A (4, 0, 0) and B (- 4, 0, 0) is equal to 10.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Prove that : $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$

OR

Prove that : $\cos^3 A + \cos^3(120° + A) + \cos^3 (240° + A) = \frac{3}{4}\cos 3A$

33. If the A.M. and G.M. between two numbers are in the ratio m : n, then prove that the numbers are in the ratio $m+\sqrt{m^2-n^2}\,;\,m-\sqrt{m^2-n^2}$.

OR

The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 and the third is increased by 1, we obtain three consecutive terms of a G.P. Find the numbers.

34. Differentiate the following functions :

(i) $\frac{x^2-x+1}{x^2+x+1}$

(ii) $\frac{x\tan x}{\sec x+\tan x}$

35. Calculate the mean and standard deviation of the following distribution:

Class-interval:	31–35	36–40	41–45	46–50	51–55	56–60	61–65	66–70
Frequency:	2	3	8	12	16	5	2	3

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Dr Jitender Singh, Lecturer Physical Education is selecting Students for volley ball team for inter zonal sports competitions. 6 students are required for team and there are total 15 students (8 Boys and 7 Girls) available. Two students named Anurag and Rakhi are very good in every sports. Based on above information, answer the following questions:
    i) In how many ways this selection will be done, if Anurag and Rakhi will be already selected?
    ii) In how many ways this selection will be done, if there will be no Girl in team?
    iii) In how many ways this selection will be done, if there is a condition of 3 Boys and 3 Girls in Team?

    OR

    In how many ways this selection will be done, if Anurag and Rakhi will not be already selected?

37. A person is standing at a point A of a triangular park ABC whose vertices are A(2, 0), B(3, 4) and C(5, 6). Based on the above information answer the following :-
    I. He wants to reach BC in least time. Find the equation of the path he should follow.
    II. Find the shortest distance travelled by him to reach BC.
    III. Suppose he meets BC at a point D. Find the coordiantors of the point D

    OR

    Find the area of the triangular park ABC.

38. To make himself self-dependent and to earn his living, a person decided to setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of Rs. 15000 per month and a cost of Rs. 30 per unit to manufacture.
    Based on the above information answer the following :-
    I. If x units of hand sanitizers are manufactured per month. What is the profit function?
    II. What is the monthly cost borne by the person if he decided to manufacture 1500 units in a month?

SAMPLE PAPER-1

ANSWERS

1. A 2. C 3. C 4. A 5. C 6. B 7. B 8. B 9. C 10. A 11. A 12. B 13. B
14. B 15. C 16. D 17. C 18. B 19. C 20. B

22. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

23. (-1,3)

24. S = {H, TH, TTH, TTHH, TTTTH, ….}

25. 2/5 OR 0.6

26. 1170

27. $\theta = n\pi$, $n \in Z$

28. Between 30°C and 35°C

29. 171 OR $\frac{76545}{8}$

30. $x^2 + y^2 - 12x + 11 = 0$ and $x^2 + y^2 + 4x - 21 = 0$ OR $\frac{7x^2}{247} + \frac{15y^2}{247} = 1$

31. $9x^2 + 25y^2 + 25z^2 - 225 = 0$

33. 12,7,2. or 3,7,11

34. (i) $\frac{2(x^2-1)}{(x^2+x+1)^2}$ (ii) $\frac{x\sec x(\sec x - \tan x) + \tan x}{(\sec x + \tan x)}$

35. $\overline{X} = 50, \sigma = 7.62$

36. (i) 715 (ii) 28 (iii) 1960 or 1716

37. (i) x+y=2 (ii) $3/\sqrt{2}$ (iii) (1/2 , 3/2) or 3

38. (i) 15000 + 30x (ii) Rs 60000

Sample Question Paper- 2
Class XI
Session 2022-23
Mathematics (Code-041)

Time Allowed: 3 Hours | Maximum Marks: 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A
(Multiple Choice Questions)

Each question carries 1 mark

Ques.1 For any two sets A and B, $A \cap (A \cup B)'$ is equal to

(a) A (b)B

(c) Ø (d)$A \cap B$

Ques.2 The value of $\tan 1^{\circ}\ \tan 2^{\circ}\ \tan 3^{\circ} \ldots \tan 89^{\circ}$ is

(a) 0 (b) 1

(c) 1/2 (d) Not Defined

Ques.3 ${}^{43}C_{r-6} = {}^{43}C_{3r+1}$,then value of r is

(a) 12 (b) 8

(c) 6 (d) 10

Ques.4 If a+ib = c+id, then

(a) $a^2 + c^2 = 0$

(b) $b^2 + c^2 = 0$

(c) $b^2 + d^2 = 0$

(d) $a^2 + b^2 = c^2 + d^2$

Ques.5 if $f(x) = x^3 - \frac{1}{x^3}$,then $f(x) + f(\frac{1}{x})$is equal to

(a) $2x^3$

(b) $\frac{2}{x^3}$

(c) 0

(d) 1

Ques.6 The number of triangles that are formed by choosing the vertices from a set of 12 points seven of which lie on the same line is

(a) 105

(b) 15

(c) 175

(d) 185

Ques.7 The x-intercept and y-intercept of the line 5x-7=6y, respectively are

(a) $\frac{7}{5}$ and $\frac{7}{6}$

(b) $\frac{7}{5}$ and - $\frac{7}{6}$

(c) $\frac{5}{7}$ and $\frac{6}{7}$

(d) - $\frac{5}{7}$ and $\frac{6}{7}$

Ques.8 If X = $\{8^n - 7n - 1 : n \in N\}$ and

Y= $\{49n - 49 : n \in N\}$. Then

(a) X C Y

(b) Y C X

(c) X=Y

(d) XΩY= Ø

Ques.9 The total number of term in the expansion of $(x+a)^{100} + (x-a)^{100}$ after simplification is

(a) 50

(b) 202

(c) 51

(d) none of these

Ques.10 $\lim_{\theta\to 0}\frac{1-Cos\,4\theta}{1-Cos\,6\theta}$ is

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$

(c) - $\frac{1}{2}$ (d) -1

Ques.11 while shuffling a pack of 52 playing cards, 2 are accidently dropped. The chances that the missing cards be of different colours is

(a) $\frac{29}{52}$ (b) $\frac{1}{2}$

(c) $\frac{26}{51}$ (d) $\frac{27}{51}$

Ques.12 The radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is

(a) 1 (b) 2

(c) 3 (d) 5

Ques.13 The domain and range of the real function F defined by $f(x) = \frac{4-x}{x-4}$ is given by

(a) Domain = R , Range= {-1,1}

(b) Domain = R – {1}, Range= R

(c) Domain = R – {4}, Range= {-1}

(d) Domain = R – {-4} , Range= {-1,1}

Ques.14 In a leap year the probability of having 53 Sundays or 53 Mondays is

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

Ques.15 If f(x)=xsinx, then $f'(\frac{\pi}{2})$ is equal to

(a) 0 (b) 1

(c) -1 (d) $\frac{1}{2}$

Ques.16 Everday in a room shakes hands with everybody else. The total number of handshakes is 66. The total number of persons in the room is

(a) 11 (b) 12

(c) 13 (d) 14

Ques.17 The domain for which the functions definied by $f(x) = 3x^2 - 1$ and $g(x) = 3+x$ are equal is

(a) $\{ -1, \frac{4}{3} \}$ (b) $[-1, \frac{4}{3}]$

(c) $(-1, \frac{4}{3})$ (d) $[-1, \frac{4}{3})$

Ques.18 $\lim_{x \to \pi} \frac{Sin\ x}{x-\pi}$ is

(a) 1 (b) 2

(c) -1 (d) -2

ASSERTION-REASON BASED QUESTION

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answers of the following choices:

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

Ques.19 Assertion (A) : The following assignment of probabilities to each outcome are valid.

Outcome	W_1	W_2	W_3	W_4	W_5	W_6
Probability	1	0	0	-1	0	1

Reason (R) : Sum of all assigned values of probabilities should be 1.

Ques 20 Assertion (A) : The following pair of sets are equal .

A = { x : x is a letter in the word FOLLOW}

B = { y : y is a letter in the word WOLF}

Reason (R) : Two sets A and B are said to be equal if they have exactly the same elements.

Section-B

This section comprises of very short answer type questions (vsa) of 2 marks each.

Ques.21 Prove that tan3x tan2x tanx = tan3x – tan2x - tanx

OR

A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second.

Ques.22 The vertices of triangle PQR are P(2,1), Q(-2,3) and R(4,5). Find equation of the median through the vertex R.

Ques.23 Solve the inequality, $3x-5 < x+7$, when

(a) x is a whole number

(b) x is a real number

OR

A solution is to be kept between $40^{\circ}C$ and $45^{\circ}C$. What is the range of temperature in degree fahrenheit, if the conversion formula is $F=\frac{9}{5}C+32$?

Ques.24 Prove the following :- $\text{Cos}4x = 1 - 8\sin^2 x\cos^2 x$

Ques.25 Find the points on the x-axis, whose distance from the lines $\frac{x}{3}+\frac{y}{4}=1$ are 4 units.

Section C

(This section comprises of short answer type questions (SA) of 3 marks each)

Ques.26 Let P={3,4,5,{1,2}}, which of following satatement are true and which are false?

(i) $\{1,2\} \in P$

(ii) $\{3,4,5\} \in P$

(iii) $\emptyset \in P$

OR

If U ={1,2,3,4,5,6,7,8,9,10}, A= {1,2,3,5}, B= {2,4,6,7} and C= {2,3,4,8};find

(i) $(C-A)'$

(ii) $(B\cup C)'$

(iii) $(A\cap B\cap C)'$

Ques.27 Show that points A(4,-3,-1) , B(5,-7,6) and C (3,1,-8) are collinear.

Ques.28 Find $(x+1)^6 + (x-1)^6$. Hence or otherwise evaluate $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

OR

Expand using Binomial theorem $(1 + \frac{x}{2} - \frac{2}{x})^4$, $x \neq 0$

Ques.29 Find the equation of the ellipse , with major axis along the x axis and passing through the points (4,3) and (-1,4).

OR

Find the equation of the hyperbola where foci are $(0, \pm 12)$ and the length of latus rectum is 36.

Ques 30 The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

Ques 31 If $(x + i y)^3 = u+iv$ Where $u , v, x, y \in R$, then show that

$$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$$

Section D

(This section comprises of long answer – type question (LA) of 5 marks each)

Ques 32 Find the mean and variance of the following frequency distribution :-

Classes	0-10	10-20	20-30	30-40	40-50
Frequencies	5	8	15	16	6

OR

The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observation are 1,2 and 6 , find the other two observations.

Ques 33 Find the value of $\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\cos\frac{4\pi}{5}\cos\frac{8\pi}{5}$

OR

Prove that $\cos^2 x+\cos^2(x+\frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3}) =\frac{3}{2}$

Ques 34 If a and b are the roots of $x^2 - 3x + p = 0$ and c , d are roots of $x^2 - 12x + q = 0$, where a,b,c ,d form a G.P . Prove that (q+p) : (q-p) = 17:15

Ques 35 Find the derivative of following functions (where m,n,a,b,c,d are fixed non – zero constants)

(i) $(ax+b)^n (cx+d)^m$

(ii) $\frac{X}{\sin^n x}$

Section-E

(This section comprises of 3 case study/ passage- based questions of 4 marks each. Internal choice is provided in Q 36 and Q 38).

Ques.36 In a survey of 600 students in a school, 150 students liked Tennis and 225 liked cricket, 100 students liked both tennis and cricket. One student is chosen at random.Based on the above information answer any four of the following questions:-

(i) Find the probability that the student liked tennis or cricket

(a) $\frac{11}{24}$ (b) $\frac{5}{12}$

(c) $\frac{19}{24}$ (d) $\frac{1}{2}$

(ii) Find the probability that the student neither liked tennis nor cricket

(a) $\frac{7}{12}$ (b) $\frac{13}{24}$

(c) $\frac{5}{24}$ (d) $\frac{1}{2}$

(iii) Find the probability that the student liked tennis but not cricket

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$

(c) $\frac{1}{12}$ (d) $\frac{3}{4}$

(iv) Find the probability that the student liked cricket only

(a) $\frac{1}{24}$ (b) $\frac{9}{24}$

(c) $\frac{7}{24}$ (d) $\frac{5}{24}$

(v) Find how many students neither liked tennis nor cricket

(a) 325 (b) 125

(c) 225 (d) None of these

Ques.37 A class consists of 5 boys and 5 girls. The class teacher wants to arrange them in a line in different ways. Based on the above information answer the following questions:-

(i) Find the number of ways in which boys and girls sit alternatively

(a) $(5!)^2 \times (5!)^2$ (b) 5! 6!

(c) $2 \times 5!$ (d) $(5!)^2 + (5!)^2$

(ii) Find the number of ways in which no two girls sit together

(a) $5! \times 5!$ (b) $6! \times 6!$

(c) $5! \times 6!$ (d) $(5!) \times 4!$

(iii) Find the number of ways in which all the girls sit together

(a) 6!6! (b) 2!5!5!

(c) 5!5! (d) 6!5!

(iv) Find the number of ways in which all the girls are never together

(a) $10! - 6!\,6!$ (b) $10! - 5!\,6!$

(c) $10! - 5!\,5!$ (d) None of these

Ques.38 To make himself self-dependent and to earn his living, a person decided to setup a small scale business of manufacturing hand sanitizers. He estimated a fixed cost of ₹15000 per month and a cost of ₹30 per units to manufacture. Based on the above information, answer any four of the following: -

(i) If x units of hand sanitizers are manufactured per month, what is the cost function?

(a) 15000 – 30x (b) 15000 + 30x

(c) 15000 + x (d) 15000 + 31x

(ii) If each unit is sold for ₹45.What is the selling (revenue) function?

(a) 30x (b) 45 + x

(c) 45x (d) 45 + 30x

(iii) What is the profit function?

(a) 15x + 15000 (b) 15 (x – 1000)

(c) 15x (d) None of these

(iv) For no profit, no loss situation in a month, how many units should be manufactured and sold?

(a) 500 (b) 750

(c) 1000 (d) 1500

(v) What is the monthly cost borne by the person if he decided to manufacture 1500 units in a month?

(a) 15000 (b) 30000

(c) 45000 (d) 60000

SAMPLE PAPER-2

ANSWER

ANS-1 (c) Ø

ANS-2 (b) 1

ANS-3 (a) 12

ANS-4 (d) $a^2 + b^2 = c^2 + d^2$

ANS-5 (c) 0

ANS-6 (d) 185

ANS-7 (b) $\frac{7}{5}$ and - $\frac{7}{6}$

ANS-8 (a) X C Y

ANS-9 (c) 51

ANS-10 (a) $\frac{4}{9}$

ANS-11 (c) $\frac{26}{51}$

ANS-12 (d) 5

ANS-13 (c) Domain = R – {4}, Range= {-1}

ANS-14 (b) $\frac{3}{7}$

ANS-15 (b) 1

ANS-16 (b) 12

ANS-17 (a) $\{ -1, \frac{4}{3} \}$

ANS-18 (c) -1

ANS-19 (d) A is false but R is true

ANS-20 (a)

ANS-21 OR

12 πradian

ANS-22 $3x - 4y + 8 = 0$

ANS-23 (a) {0,1,2,3,4,5}

(b) {x : x ∈ R and x < 6 }

OR

104°F to 113°F

ANS-25 (-2, 0) and (8,0)

ANS-26 (i) True

(ii) True

(iii) False

OR

(i) {1,2,3,5,6,7,9,10}

(ii) {1,5,9,10}

(iii) {1,3,4,5,6,7,8,9,10}

ANS-28 $(x + 1)^6 + (x - 1)^6 = 2x^6 + 30x^4 + 30x^2 + 2$

$(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 = 198$

OR

$\frac{16}{x^4} - \frac{32}{x^3} + \frac{8}{x^2} + \frac{16}{x} - 5 - 4x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{16}x^4$

ANS-29 $7x^2 + 15y^2 = 247$

OR

$\frac{y^2}{36} - \frac{x^2}{108} = 1$ i.e. $3y^2 - x^2 = 108$

ANS-30 Minimum length is 41 cm

ANS-32 Mean = 27

Variance = 132

OR

4 and 9

ANS-33 $-\frac{1}{16}$

ANS-35 (i) $(ax + b)^{n-1} (cx + d)^{m-1} \{(m+n)acx + mcb + nad\}$

(ii) $\frac{\sin x - nx\cos x}{(\sin x)^{n+1}}$

ANS-36

(i) (a)

(ii) (b)

(iii) (c)

(iv) (d)

(v) (a)

ANS-37

(i) (d)

(ii) (c)

(iii) (d)

(iv) (d)

ANS-38

(i) (b)

(ii) (c)

(iii) (b)

(iv) (c)

(v) (d)

Sample Question Paper- 3
Class XI
Session 2022-23
Mathematics (Code-041)

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A
(Multiple Choice Questions)

Each question carries 1 mark

Ques.1 If $f(x)=\sqrt{x}+\frac{1}{x}$, then $f'(4)$ =

(a) $\frac{3}{4}$ (b) $\frac{9}{4}$

(c) $\frac{3}{16}$ (d) $\frac{9}{16}$

Ques.2 Eccentricity of equilateral hyperbola is

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 2 (d) $\frac{1}{2}$

Ques.3 $\lim_{x\to 0} \frac{e^{3x}-1}{x}$ is equal to

(a) $\frac{1}{3}$ (b) 3

(c) $\frac{1}{9}$ (d) 9

Ques.4 If A and B are two sets, then $A \cap (A \cap B)' =$

(a) $A \cap B$ (b) $A' \cap B$

(c) $A \cap B'$ (d) $A' \cap B'$

Ques.5 If $A = \{x : x \in R, x>4\}$ and $B = \{x : x \in R, x \leq 5\}$ then $(A \cap B) =$

(a) (4,5) (b) [4,5]

(c) [4,5) (d) (4,5]

Ques.6 If A = {1,2,3}, B ={1,4,6,9} and R is a relation from A to B defined by x is less than y. The range of R is

(a) Ø (b) {1}

(c) {4,6,9} (d) {1,4,6,9}

Ques.7 For any sets A and B ; $(A \cap B) \cup (A - B) =$

(a) A (b) B

(c) A' (d) B'

Ques.8 If $x \in R$, range of $f(x) = \frac{1}{1+x^2}$ is

(a) R (b) $(0,\infty)$

(c) (0,1) (d) (0,1]

Ques.9 Let A= {1,2}, B = {3,4}. The number of subsets of A×B is

(a)4 (b) 8

(c) 16 (d) 32

Ques.10 $\frac{\cos 10°+\sin 10°}{\cos 10°-\sin 10°}=$

(a) $\cot 35^0$ (b) $\tan 35^0$

(c) $-\cot 35^0$ (d) $-\tan 35^0$

Ques.11 $3i^{15} - 5i^8 + 1$ represents the following complex number

(a) 3i - 5 (b) -3i - 5

(c) 3i - 4 (d) -3i - 4

Ques.12 In how many ways can we form a four digit number using all the given digits 2,3,4,2.

(a) 24 (b) 12

(c) 8 (d) 4

Ques.13 Find n, if ${}^nC_5 = {}^{10}C_4 + {}^{10}C_5$

(a) 4 (b) 5

(c) 9 (d) 11

Ques.14 The total number of terms in the expansion of

$(x + 11)^{23} - (x - 11)^{23}$ after simplification is

(a) 0 (b) 12

(c) 24 (d) 46

Ques.15 Distance of the point (7,-3,5) from its reflection in XZ plane is

(a) 6 units (b) 25 units

(c) 49 units (d) $\sqrt{83}$ units

Ques.16 Derivation of sinxcosx with respect to x is

(a) cosec2x (b)sec2x

(c)cos2x (d)sin2x

Ques.17 For any two events A and B, if P(A)= 0.05,P(B)= 0.10 and P(A∩B) = 0.02, find $P(\bar{A} \cap \bar{B})$

(a) 0.98 (b) 0.83

(c) 0.90 (d) 0.85

Ques.18 The probability that a leap year will have 53 Mondays or 53 Tuesdays is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

ASSERTION-REASON BASED QUESTION

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer of the following choices:

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

Ques.19 Assertion (A) : Given 4 flags of different colours, then number of different signals can be generated, if a signal requires the use of 2 flags one below the other is 12.

Reason (R) : If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is m × n.

Ques 20 Assertion (A) : If E_1 and E_2 are two mutually exclusive events, then $E_1 \cap E_2 = \emptyset$

Reason (R) : If E_1 and E_2 are mutually exclusive and exhaustive events, then $E_1 \cap E_2 = \emptyset$ and $E_1 \cup E_2 = S$.

Section-B

This section comprises of very short answer type questions (VSA) of 2 marks each.

Ques.21 A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second.

Ques.22 Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

OR

Find value of $\tan\frac{\pi}{8}$

Ques.23 A solution is to be kept between 40^0C and 45^0C. What is the range of temperature in degree fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

Ques.24 Four cards from a pack of 52 cards are drawn at random. Find the probability that all four cards are of same suit.

Ques.25A card is drawnfrom a deck of 52 cards. Find the probability of getting a Jack or a King or a spade card.

Section-C

(This section comprises of short answer type questions (SA) of 3 marks each)

Ques.26 If U={1,2,3,4,5,6,7,8,9,10}, A ={ x:x is prime}, B= {x:x is even integer}. Write

(i) A - B

(ii) A $\cap$B$'$

OR

Using Venn diagram, prove that $(A \cup B)' = A' \cap B'$

Ques.27 Find real θ such that $\frac{3+2i\ sin\theta}{1-2i\ sin\theta}$ is purely imaginary;

$0\square \leq \theta \leq 90°$

OR

If z = x – iy and $z^{1/3}$ = p + iq, then find the value of:

$$\frac{\frac{x}{p} + \frac{y}{q}}{p^2 + q^2}$$

Ques.28The temprerature (in celsius) in a city is considered normal when the average of three daily measurements (morning, afternoon, night) is between 19.2 and 29.8. On one day, if morning and night temperature (in celsius) are 19.48 and 19.85 respectively, find the range of afternoon temperature (in celsius) that will result in normal temperature of the day.

OR

Solve the inequality $\frac{x+8}{x+2} > 2$. Represent the solution on number line.

Ques.29 Using binomial theorem , prove that $6^n - 5n$ always leaves the remainder 1 when divided by 25.

Ques 30 Find the equation of ellipse that satisfies the following conditions: Centre at origin, major axis on the y- axis and passes through the points (3,2) and (1,6).

Ques Find the equation of the set of points P, the sum of whose distances from A(0,5,0) and B(0,-5,0) is equal to 15.

Section D

(This section comprises of long answer type questions (LA) of 5 mark each)

Ques 32 Draw the graph of the following function.

$F(x) = |x-1| + |2+x|$ $for\ all - 3 \leq x \leq 3$. Also find its range

OR

Draw graph of the function

$$f(x) = \begin{cases} 1 + 2x, x < 0 \\ 3 + 5x, x \geq 0 \end{cases}$$

Also find its range.

Ques 33 If A is the arithmetic mean and G_1, G_2 be two geometric mean between any two numbers, then prove that

$$2A = \frac{G_1^2}{G_2} + \frac{G_2^2}{G_1}$$

Ques 34Evaluate $\lim_{x \to \frac{1}{2}} \left(\frac{8x-3}{2x-1} - \frac{4x^2+1}{4x^2-1} \right)$

OR

Evalute $\lim_{x \to \frac{\pi}{4}} \frac{tan^3 x - \tan x}{\cos(x + \frac{\pi}{4})}$

Ques 35 Determine the mean and standard deviation for the following distribution :

Marks	Frequency
2	1
3	6
4	6
5	8
6	8
7	2
8	2
9	3
10	0
11	2
12	1
13	0
14	0
15	0
16	1

Section-E

(This section comprises of 3 case study/ passage- based questions of 4 marks each.)

Ques.36 Daksh travelled from Delhi to Mumbai with his suitcase. After reaching to Mumbai, he forgot the password to unlock the suitcase. He remembered that there are 2 digits followed by 2 letters in the password.

(i) Find total number of possibilites for the password, if repetition is allowed. (1)

(ii) How many possibilities are there for the password, if neither numbers nor words are repeated. (1)

(iii) How many different passwords are possible, if Daksh remembered first two and last entry, and repetition is allowed. (2)

OR

How many different passwords are possible, if Daksh remembered first and last entry and no entry repeated? (2)

Ques.37 A teacher mentioned in his class that there is some $\theta \epsilon R$ such that $\sec\theta = \frac{-13}{12}$, where θ lies in second quadrant. Then he asks his students about some other values.

(i) What will be the value of $\tan\theta$

(a) $\frac{5}{12}$ (b) $\frac{-5}{12}$

(c) $\frac{5}{13}$ (d) $\frac{-5}{13}$

(ii) What will be the value of $\cos\frac{\theta}{2}$

(a) $\frac{5}{\sqrt{26}}$ (b) $\frac{-5}{\sqrt{26}}$

(c) $\frac{1}{\sqrt{26}}$ (d) $\frac{-1}{\sqrt{26}}$

OR

What will be the value of $\sin\frac{\theta}{2}$

(a) $\frac{5}{\sqrt{26}}$

(b) $\frac{-5}{\sqrt{26}}$

(c) $\frac{1}{\sqrt{26}}$

(d) $\frac{-1}{\sqrt{26}}$

(iii) Find value of $\sin 2\theta$

(a) $\frac{5}{13}$

(b) $\frac{-5}{13}$

(c) $\frac{120}{169}$

(d) $\frac{-120}{169}$

Ques.38 Nikhil and his friend Aman live in a neat and clean society in a big city. We imagine cartesian axes in the society placing central park at the origin. Nikhil's house lies on a straight lane represented by straight line $4x + 7y + 5 = 0$. Aman's house lies on another straight lane represented by straight line $2x – y = 0$.

(i) Nikhil and Aman can meet at the intersection of two lanes . Find the coordinates of the point of their meeting junction. (2)

(ii) Aman is in his room. If he starts from a point in his room having coordinates (1,2) and moves along the straight lane along his house $2x – y = 0$, find distance covered by him to reach the meeting junction.(2)

SAMPLE PAPER-3

ANSWER

ANS-1 (c) $\frac{3}{16}$

ANS-2 (a) $\sqrt{2}$

ANS-3 (b) 3

ANS-4 (c) $A \cap B'$

ANS-5 (d) $(4,5]$

ANS-6 (c) $\{4,6,9\}$

ANS-7 (a) A

ANS-8 (d) $(0,1]$

ANS-9 (c) 16

ANS-10 (a) $\cot 35^0$

ANS-11 (d) $-3i - 4$

ANS-12 (b) 12

ANS-13 (d) 11

ANS-14 (b) 12

ANS-15 (a) 6 units

ANS-16 (c) $\cos 2x$

ANS-17 (b) 0.83

ANS-18 (c) $\frac{3}{7}$

ANS-19 (a)

ANS-20 (b)

ANS-21 12π

ANS-22 OR

$\sqrt{2} - 1$

ANS-23 104°F to 113°F

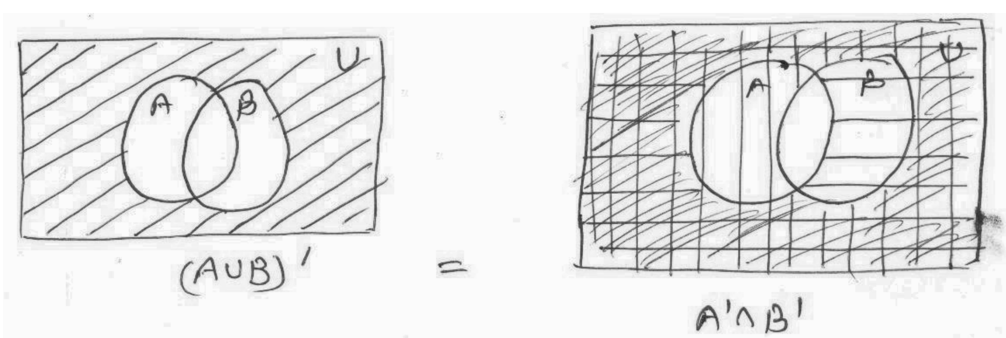
ANS-24 $\frac{44}{4165}$

ANS-25 $\frac{4}{52} + \frac{4}{52} - \frac{2}{52} = \frac{3}{26}$

ANS-26 (i) {3,5,7}

(ii) {3,5,7}

OR



ANS-27 $\theta = 60°$

OR

-2

ANS-28 18.27 < x <40.07

OR

$x \in (-2,4)$

ANS-30 $\frac{x^2}{10} + \frac{y^2}{40} = 1$

ANS-31 $36x^2 + 20y^2 + 36z^2 - 1125 = 0$

ANS-32 Range [5,7]

OR

Range $(-\infty,1) \cup [3,\infty)$

ANS-34 $\lim_{x\to\frac{1}{2}} \left(\frac{2(3x+2)}{2x-1}\right) = \frac{7}{2}$

OR

-4

ANS-35 Mean $\bar{x}$ = 5.975≈ 6, Standard Deviation ≈ 2.85

ANS-36 (i) 67600

(ii) 58500

(iii) 26

OR

(iii) 225

ANS-37 (i) (b) $\frac{-5}{12}$

(ii) (c) $\frac{1}{\sqrt{26}}$

OR

(ii) (a) $\frac{5}{\sqrt{26}}$

(iv) (d) $\frac{-120}{169}$

ANS-38 (i) $(\frac{-5}{18}, \frac{-5}{9})$

(ii) $\frac{23}{18}\sqrt{5}$ units

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